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AIR-CUSHION-SUPPORTED VEHICLE FAN DYNAMIC RESPONSE

A REVIEW OF THE LITERATURE

David W. Taylor Naval Ship Research and
Development Center, Bethesda, Maryland
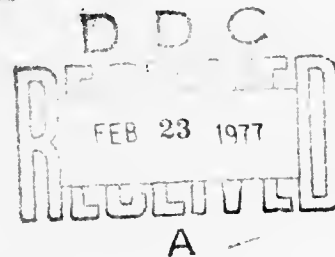
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DAVID W. TAYLOR NAVAL SHIP
RESEARCH AND DEVELOPMENT CENTER

Bethesda, Md.   20084

AIR-CUSHION-SUPPORTED VEHICLE FAN DYNAMIC RESPONSE:

A REVIEW OF THE LITERATURE

by

David D. Moran

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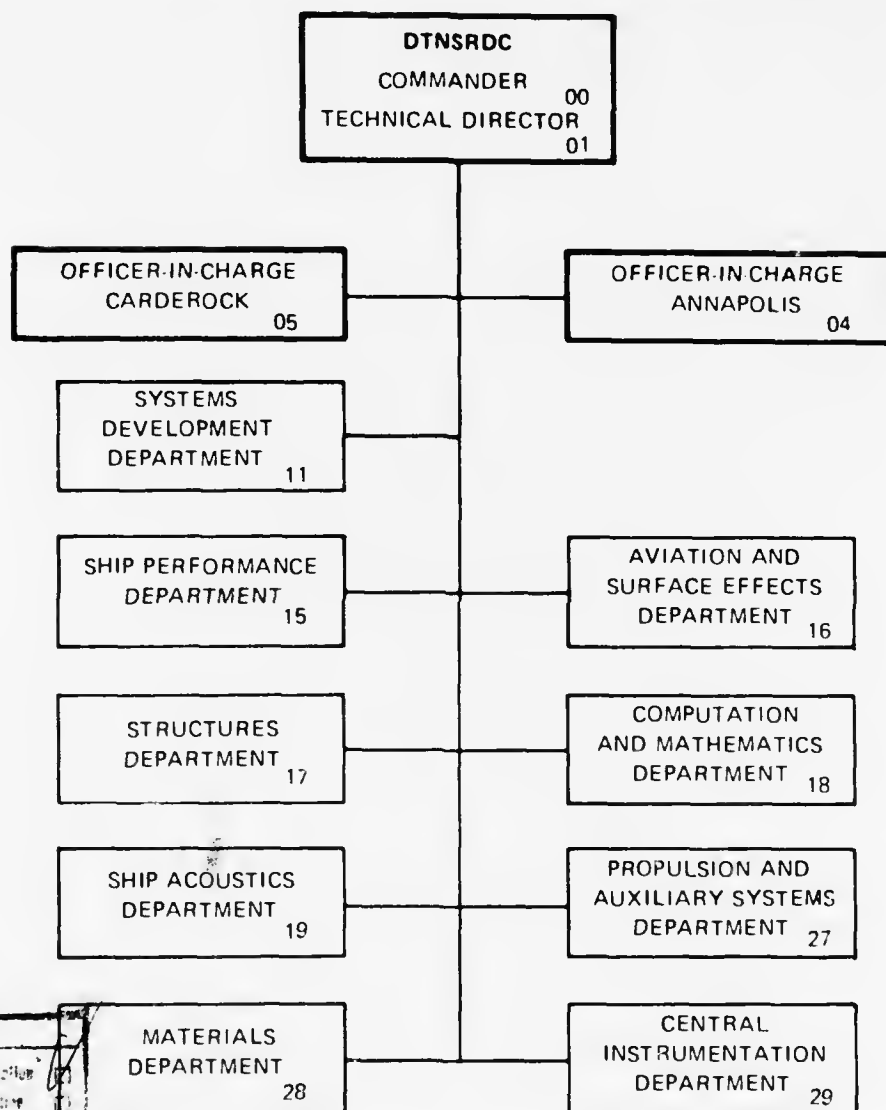FEB 23 1977

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    The mechanisms which constitute the dynamic response of air-cushion-supported vehicle lift fans subjected to unsteady excitation are summarized and examined through a review of research as reported in literature published between 1932 and 1976.

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NOTATION

$\bar{b}$	Slope of blower characteristic curve
C	Chord length
C_f	Frequency coefficient
dp/dq	Slope of fan characteristic or fan map
f	Frequency of forced fluctuation
G	Characteristic function, Equation (3)
$\bar{g}$	Slope of valve characteristic curve
m'	Suction pipe length
N	Sum of suction pipe and delivery pipe lengths
N_R	Number of blades
n	Rotational speed of impeller
n'	Delivery pipe length
p	Fan pressure
q	Fan discharge
t	Time
u	Velocity of air in pipeline
V	Free stream velocity
α	Damping coefficient determined by compressor and system characteristics
$\dot{\alpha}^{(s)}$	Rate of change of angle of attack of airfoil evaluated at instant that dynamic stall occurs
ΔW	Variation of weight flow from initial or stall value
ΔW_s	Weight-flow increment between equilibrium points on upper and lower branches of compressor characteristic
Λ	Constant determined by compressor and system characteristics

NOTATION (Continued)

λ_R Stagger angle of cascade

ϕ Flow coefficient

Ω Undamped natural frequency of compressor and receiver

ω System constant that determines oscillation frequency

ABSTRACT

The mechanisms which constitute the dynamic response of air-cushion-supported vehicle lift fans subjected to unsteady excitation are summarized and examined through a review of research as reported in literature published between 1932 and 1976.

ADMINISTRATIVE INFORMATION

INTRODUCTION

The air-cushion-supported vehicle (ACV) or surface-effect ship
(SES) is a new and unique concept in several respects. The system com-
ponents, which constitute the ACV/SES design, function together to produce
a dynamic response to sea-wave excitation requiring study and investi-
gation different from other types of ships.

Three unique characteristics of air-cushion-supported craft pre-
cipitated their design and construction. The first is the very low or,
indeed, zero, hull resistance which these craft exhibit. Sidewall type
surface effect vehicles can be designed to optimize surface-piercing hull
drag. The wave-making drag of the cushion pressure distribution is amen-
able to a degree of control and optimization. In general, the ACV/SES is
capable of a greater design speed than a displacement ship.

The second characteristic of some surface-effect vehicles
is an amphibious capability. The benefits of amphibious high-speed
vehicles are obvious for both military and commercial uses. The British
have indeed made a commercial success (Hoverloyd) of full-skirted ACV
ferry operations across the English Channel. Although the amphibious
nature of the craft is not required in this specific case, it does make
the operations of loading and unloading easier. The United States Navy is
presently constructing two prototype air-cushion vehicles which will serve
as evaluation craft for ACV implementation in amphibious assault warfare.

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In this application, the amphibious capabilities will be exploited fully
since the craft will operate over water, overland, in surf zones and upon
the decks of support ships.

The third characteristic of air-cushion ships which has been noted
historically as a major reason to seriously consider ACV implementation is
their seakeeping response. The ACV from its inception was expected to be
a good seakeeper. An early expectation was that the vehicle would ride
over the waves on its air-cushion shock absorber as if the sea were nearly
flat. This expectation could not of course hold up under either
dynamic analysis or actual ACV testing. The surface-effect vehicle is a
dynamic system and like all dynamic systems will exhibit response which
is a function of the frequency and amplitude of the excitation. The excita-
tion is, in this case, the wave field or overland terrain over which the
vehicle travels. Air-cushion-supported vehicles do have a great promise
as good seakeepers, but the details of their behavior in a seaway must be
known in order to make use of their benefits and minimize their disadvan-
tages.

The dynamic response of the ACV may be examined in a number of
ways. A total-system or black-box approach is the fastest, easiest, and
most convenient. This technique does not, however, allow parametric
analysis or performance improvement identification. Empirical analysis
of the ACV produces the most reliable dynamic performance criteria on
model scale, but the technique suffers from unresolved questions of scaling
of dynamic performance. An alternative to the total-system approach is

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interactive systems component analysis. This method allows a great flex-
ibility in design and evaluation of ACV motions but is limited by cumula-
tive errors produced by inaccurate mathematical models of physical systems.
When the separate models are combined in a series-constructed system, the
individual component errors are multiplicative. Nonetheless, this is a
reasonable and attractive approach. The following presentation relates to
one specific element of the system, the lift system.

The ACV lift system consists of the compressor, the plenum, the
bow and stern seals, and the airflow control devices. The principal
requirement of the lift system in calm water is to provide the required
pressure to support the ship. The principal requirements in a seaway are
to supply sufficient airflow variations at minimum pressure fluctuations,
to accommodate changes in cushion volume due to the passage of waves, and
to control the motions of the ship. All types of lift system fans are
available from centrifugal flow to axial flow machines with or without
variable geometry. Airflow can be varied and motions modified by varying
the fan flow (through changes in fan speed or design), or by varying the
flow through a cushion vent when using a fixed geometry fan. The dynamic
response of an air cushion vehicle is determined by the total lift system
response including fan characteristics, seal dynamics, and other lift
system characteristics.

The lift fan is the most important feature of the system. Properly
designed, the lift fan must have a proven capability of functioning under
conditions of varying load. This dynamic capability is one feature of

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the system which has been somewhat neglected until recently. The dynamic

performance of an ACV lift fan can differ dramatically from its static perfor-

mance. Examples (Durkin (1974),*Thomas (1972)) of this behavior are shown

in Figures 1 and 2 which give the pressure-discharge relationships for

both static and dynamic performance of axial flow fans. The performance

indicated in Figure 1 was obtained from a dynamic fan-testing device which

is a dynamic system itself and, therefore, the indicated behavior cannot

be accepted as purely the result of the fan characteristics. The figure

does show dynamic behavior which is indicative of that observed in ACV

lift systems. The major point of this discussion is that the lift fan is

a dynamic component and must be analysed as such in the complete system.

The example shown in Figure 2 is obtained from the vertical oscillation of

the ground plane under an ACV model and therefore also represents a total

system response.

Another example of dramatic dynamic effects in an ACV lift system

was the appearance of backflow in the lift fans of a high length-to-beam

ratio surface-effect ship model (XR-5) tested in regular and irregular

waves by Moran (1975) and Ricci and Moran (1976). In an experiment directed

toward an examination of this specific problem It was observed that regular

wave excitation produced conditions of periodic reverse flow through the

lift fans. The reverse flow persisted for approximately 10 percent of the

fan flow oscillation period.

The time-dependent character of the fan discharge was also measured

by Moran (1976a) for a fully skirted ACV (AALC-JEFF(B)) in a study of the

*References begin on page 64 and are listed in alphabetical order by the
first author's surname.

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overland dynamic response to regular wave excitation. Although these
data did not indicate reverse flow conditions the magnitude of discharge
variations over an excitation period was significant. A similar experi-
mental study was performed by Moran (1976b) for a surface-effect ship model
in a wave field. These results also show that the dynamic performance of
the lift fan for an air-cushion-supported ship cannot be neglected. It is
therefore vital to the continued technological development of ACV dynamics
to construct a valid mathematical model of the physical mechanism involved
in the unsteady operation of lift fans.

A dynamic model for the lift system must include several mech-
anisms for system response hysteresis. In the fan itself, blade stall due
to time-dependent changes in the angle of attack can be experienced. The
most dramatic example of blade stall is associated with full flow reversal
in the fan. Stall can occur over the whole impeller disk or propagate
around the disk in an unsteady manner not directly related to the excitation
or rotational frequencies. Flow surge is another problem that can affect
the fan system, including the fan volute. This phenomenon can result from
volume resonance or conditions of volume variation in the plenum or from
the hydraulic surge generated by compressible flow in the lift system duct
work. A third hysteresis-generating mechanism is a Coanda phenomenon
resulting in nonlinear behavior of the flow under an ACV skirt. This mech-
anism is most evident for rigid seal (sidewall) or air-seal craft (peripheral
jet ground-effect machines). Two further mechanisms that give rise to
nonlinear behavior are flow separation in the fan and duct work and

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deformations of the skirt and bags.

This report is concerned only with the dynamic response of the lift system of an air-cushion-supported vehicle, with concentration on the problems of stall and surge in the lift fan. The remaining mechanisms for hysteresis are touched on as they appear in the literature review. The review presented here is arranged in chronological order according to publication date. Reference to each reviewed work is made by year of publication. A list of references and other bibliographical material, arranged in alphabetical order by the first author's surname, can be found beginning on page 64. Although a diversity of themes runs through the review, the chronological arrangement was chosen to aid the reader in an appreciation of the systematic development of fan-dynamics technology. Certain themes noted in the review are not directly applicable to the ACV/SES dynamics problem (e.g., diesel supercharger surging at very high frequencies, and helicopter rotor lift dynamics) but are included in the present discussion for their importance in the developing technology base for general fluid flow devices.

LITERATURE REVIEW

In an early study of the surging phenomena in superchargers for aircraft engines, Brooke (1932) performed a series of experiments to investigate the effect of impressed flow periodicity on the magnitude of surge. For this purpose the author fitted a rotary interrupter valve to the supercharger delivery. This enabled the outlet area to be varied periodically by a definite amplitude over a range of high frequencies. The author concluded that there is no evidence of any important effect upon surging as a result of the fluctuation of flow imposed by a high frequency variation of area of the compressor outlet. Unfortunately for present applications the author was interested only in the very high frequency excitation characteristic of that corresponding to engine valve frequencies (e.g., frequencies greater than 1000 Hz). The interrupter valve was designed to produce variations of only plus or minus 11 percent from the mean flow area. Although this is also pertinent to aircraft engine design it is not applicable to the present problem.

Flow instabilities are observed in proximity to the surge point using the equipment and techniques described above, but at flows immediately beyond the region affected by the hysteresis in recovery of delivery pressure there is no observed tendency for surging to be developed by the fluctuation of flow imposed by operating the interrupter. The natural surge point is determined in the author's experiments by progressively reducing the mass flow of air by restricting (statically) the outlet from

the supercharger. The pressure at the delivery point increases as the mass flow diminishes until at the surge point the discharge pressure decreases abruptly. The magnitude and frequency of the surging phenomenon are greatly influenced by the volume of air enclosed in the pipe system between the compressor and the throttle valve. Increasing the capacity or volume of the system causes an increase in the amplitude of pressure variation and a decrease in the resonant frequency.

Bidard (1946 and 1950) produces a stability condition for hysteresis in the pressure-discharge curve for a harmonically-excited compressor. The author indicates convergent and divergent spiral regimes as a function of the slope of the characteristic curves for the compressor and the receiving tank (or valve).

Fujii (1948) has contributed a relaxation oscillation analysis for surging in centrifugal water pump systems containing a free surface. The author's analysis is not applicable to present considerations because of the inertial and damping characteristics of the gravitationally affected components of the system (i.e. the free surface). The solution is enlightening, however, in its illustration of the convergent and divergent characterizations of the relaxation solution.

In an early paper concerning the surging of axial compressors, Pearson and Bowmer (1949) have constructed analog resistance/inductance/capacitance (RLC) circuits to describe the surging behavior of multistage axial flow compressors. Their goal is not to explain the mechanism of surging but simply to describe the compressor output response.

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The differential equation describing the analogous RLC circuit
is developed by the authors and is shown to have three possible solutions
described as

      (1)   Critically damped, no oscillators.

      (2)   Exponentially damped oscillators.

      (3)   A singular solution.

In each case an exponential decay factor includes the system resistance
or slope of the characteristic curve (dp/dq) which is negative for stalled
conditions leading to critical divergence (solution 1) or rapidly growing
oscillations (solution 2) as a system response. The authors also discuss
a continuous flow model (quasi-steady in formulation) which predicts surge
at points on the compressor characteristic curve at which the ratio of dis-
charge to pressure is a constant at a fixed fluid temperature. This solu-
tion indicates that surging will occur far below the peak on the pressure-
discharge characteristic curve, a result which is contrary to the experi-
mental evidence available to the authors. The diverging flow solution
predicts surge points very near the peak of the characteristic curve.
The oscillatory flow solution results in surge initiation at somewhat higher
discharges. The authors conclude from this discussion that there is some
justification therefore to assume that the surge line on the compressor
characteristic curve coincides with the peak pressure locus. They further
conclude that the oscillatory instability (solution 2) is unlikely to occur
in practice due to the nonlinear character of the system resistance (dp/dq)
which would "in effect place a heavy damping on the oscillations" once
they were initiated. Experimental observations by the authors show no

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evidence of the high frequency oscillations associated with this solution, but they have observed sudden pressure changes which only gradually returned to equilibrium. This behavior is not predicted by any of the authors' models.

Despite the limitations of the analyses presented in this paper, the authors must certainly be commended for their creative approach to a problem which was rather new to the engineering profession. That commendation must also be extended to their prophetic call for an evaluation of the dynamic performance of compressors in addition to the static performance historically studied.

Jenny (1950) has presented a complete discussion of the characteristic solution for one-dimensional compressible transient flow in his milestone paper published by the Brown Boveri Review. Prior to the development of more exact theories the author's method formed the basis of all analysis of the transient characteristic of flow machinery.

A paper by Kusama, Tsuji, and Oshida (1950) describes a natural instability in a pump-pipe system. The authors ascribe the instability to the positive slope portion of the steady state pressure-rise versus flow-coefficient curve for the pump examined. They have noted that the observed instability can be controlled by closing an exit valve for their system and then opening it according to some prescribed procedure. Unfortunately the significance of this prescribed procedure is unknown. The authors have demonstrated the existence of closed counterclockwise

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hysteresis loops in the pump performance characteristics for unsteady flow. They have measured the natural period of the resultant oscillations in pressure and have shown a linear increase of oscillation period with the rotational speed. The authors have also presented experimental evidence demonstrating an approximately linear increase of oscillation period with pipe length (i.e., the volume of the receiving element of the system) for fixed pump speeds. A hydraulic model of the system has produced a stability parameter with which the authors have shown experimental correlation.

The flow behavior in any rotational machinery can be examined in detail only through a careful analysis of the mechanics of flow of viscous compressible fluid in a rotational flow domain. Nemenyi and Prim (1951) have summarized the classes of nonviscous incompressible fluid flow formulated by Beltrami and Masotti and known by their names (i.e., Beltrami Fiow and Masotti Flow) and have extended their te.hniques to an analysis of the rotational flow patterns of a perfect gas.

An investigation of stall and surge phenomena in axial-flow compressors has been performed by Huppert and Benser (1953). They have shown that single-stage stall is an unsteady phenomenon in which some sectors of the annulus operate at a much lower mass-flow rate than the remainder of the sectors, rotating about the compressor axis. Uniform flow exists at a point far upstream of the stall zone, and stream lines immediately upstream of the blades have to diverge to maintain continuity around the stall zone. As a result of this spillage of flow, the stall zone under-

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goes a tangential propagation relative to the rotor, at a rate generally
opposite in direction to but lower in magnitude than the rotor-blade
speed.

The authors note that two categories of stall patterns have been
observed. The first, a progressive stall, originates at the ends of the
blade and consists of multiple stall zones extending over only a portion
of the blade span. The rotor-tip angle of attack varies with flow rate
much more rapidly than the root angle of attack does, and the rotor-
blade tip stalls first while the rest of the blade remains unstalled.
The second type of stall is a root-to-tip stall that usually consists of
only one zone extending over the entire blade span. The stall angle of
attack is approached nearly simultaneously over the entire blade span,
and the resulting variations in flow rate are relatively uniform in intensity
from root to tip. There is a sharp drop in pressure ratio at instigation
of stall, in contrast to the gradual decrease in pressure ratio seen for
progressive stall. The stalled zone must be completely eliminated in
order to unstall the root-to-tip stage stall. Typical performance curves
showing pressure coefficient as a function of flow coefficient are presented
for both categories of stall in Figure 3. It can be seen that for progressive
stall the number of stall zones increases with decreasing flow on the
left side of the characteristic curve.

Data from multi-stage compressors are reported to show a progressive-
type stall at low speeds. The flow fluctuation amplitude increases in mag-
nitude through the first stages and then decreases to a small value in the

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downstream stages. The authors present the results of a stage-stacking analysis which shows that rotating stall exists over a large portion of the compressor map at low speeds but will be instigated almost simultaneously with compressor surge at high speeds.

Multi-stage compressors with large inlet and discharge tanks frequently experience violent surge, normally with a frequency of 0.5 to 2 cycles per second. For the classical type of surge in which the compressor characteristic of pressure rise is assumed to be a continuous function of weight flow for a constant compressor speed, a solution to the differential equation of flow has been obtained in previous investigations by linearizing the compressor and discharge throttle characteristics and simplifying the external system. The resulting solution is of the form

$$\Delta W = K_i e^{-\alpha t} \sin (\omega t + K_2) \qquad (1)$$

where ΔW is the variation of weight flow from equilibrium

K_i is a constant

α is the damping coefficient

t is time

ω is a function of the oscillation frequency

K_2 is a constant expressing the phase relation

The stability of the system is defined by α, with a positive value indicating a stable system.

The authors consider another kind of surge attributed to a limit cycle operation about the compressor stall point. The pressure-ratio, weight-flow characteristics that are discontinuous at stall result in the

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hysteresis effect seen in Figure 3. The discontinuity in pressure associated with stall causes a flow-rate fluctuation if normal stability requirements are satisfied. However, stall will be relieved if the compressor flow rate becomes at least as large as the stall recovery value during the transient, and the compressor will seek equilibrium at the original point of stall on the unstalled branch of its characteristic. The solution of the linearized equations for the flow variation is

$$\Delta W = -|\Delta W_S| + \left[1 + e^{-\alpha t} \left(\Lambda \sin \omega t - \cos \omega t \right) \right] \qquad (2)$$

where ΔW is the variation of weight flow from the initial or stall value

ΔW_S is the weight-flow increment between equilibrium points
on the upper and lower branches of the compressor characteristic

α is the damping coefficient determined by compressor and system characteristics

t is time

Λ is a constant determined by compressor and system characteristics

ω is a system constant that determines the oscillation frequency

This type of cyclic operating condition, known as limit-cycle surge, is caused by a step change in pressure ratio due to complete compressor stall and will result in violent oscillations of flow and pressure ratio. The receiver pressure change will lag the flow change during the transient because of inertia and capacitance effects of the system. The authors report that oscillation amplitudes increase with decreasing α/Ω ratios, where Ω is the undamped natural frequency of the compressor and receiver. If the receiver and throttle have a large time constant, the damping coefficient

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α will be small and the compressor will probably surge. Limit-cycle surge is more likely to occur at high compressor speeds than at low speeds because of the increased slope of the throttle curve and the narrow range of weight flows for which the two branches overlap. The classical type of surge is also encountered more frequently at high speeds.

Sears (1953) has shown that the asymmetrical flow in an axial flow compressor stage exhibits rotational flow in the wake of the rotor disk. The author has presented a steady flow analysis leading to relationships between the circumferential circulation distribution and the induced velocities in the axial flow disk.

The propagation of stall in axial-flow compressors has also been investigated in Iura and Rannie (1954). The experimental results presented by the authors cover an extensive range yielding relationships between stall frequencies and fan rotational rate, stall patterns, and the extent of stall in an axial-flow compressor.

The authors have first defined the scope of their results by differentiating between the phenomena of surge and propagating stall. Surging is defined as the phenomenon in which the net flow through the entire compressor flow disk fluctuates with time. Propagating stall is defined by the authors as a phenomenon in which the total flow rate through the annulus does not vary with time. Although the mechanisms for these two phenomena are different, the authors have

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nonetheless provided an external or observational means of classifying
unsteady flow effects. The two phenomena may be coupled with respect
to the possibility of stall-exciting surge, or surging which involves
an oscillation from one propagating-stall pattern to another.

Iura and Rannie propose the same explanation for the occurrence
of propagating stall as offered by other investigators, namely that
the propagating stall is explained qualitatively as a successive
unstalling and stalling of blades in a cascade. When a particular
blade has reached a condition of stall it manifests a high resistance
to flow through the cascade and the approaching flow streamlines are
distorted such that the preceding foil encounters flow at a decreased
angle of attack whereas the following foil is subject to flow at a
large angle of attack. Flow separation is therefore inhibited on the
preceding foil and encouraged on the following fan blade. The stalled
region therefore moves in the direction of the following blades and
the frequency of stall rotation in a rotational fan may be expected
to be less than the rotational speed of the fan. The authors have
found that the stall frequency (or rotational speed) varies between
25 percent and 30 percent of the fan speed for the device tested. The
ratio of stall-to-fan frequencies changes only slightly with fan discharge.

The form and frequency of stall are dependent upon the
velocity of propagation of the disturbance from one blade to the next.
Various authors have proposed analytic models to describe this
reaction. As examples, Sears (1953) postulated a phase lag between
the fluctuating lift and the mean angle of attack of the foil for a

two-dimensional cascade. In this model the number of active blades was assumed to be infinite. In a similar analytic treatment of a two-dimensional cascade containing a finite number of blades, Marble (1953) proposed a nonlinear relationship between lift and angle of attack in a manner that forced a hysteresis loop response.

As noted by Iura and Rannie, each of the aforementioned models are simplified analytical relations which approximate in different ways the known behavior of single airfoils in stall. The present authors express the opinion that a more realistic model of the unsteady cascade flow is obtained by examining the exit flow angle and the total pressure loss through the cascade. The authors have observed that the exit flow angle can be taken as a constant but the total pressure loss is grossly nonlinear. The loss in total pressure is small and nearly constant over an angle of incidence in the range of 8 to 10 degrees but rises sharply to a high value outside of this range. It is this nonlinear behavior which, the authors propose, fixes the amplitude of the stall oscillation.

Returning to the velocity of disturbance propagation, it has appeared to be intuitive that the velocity must be inversely proportional to the time required to establish a complete separated-flow region on a foil after a sudden increase in the angle of the incident flow. The phase angle between the unsteady lift response and the angle of attack of an oscillating foil is consistent with that explanation. Further, the phase angle has been found to be rather insensitive to excitation frequency. The results of compressor

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experiments do not however show as good a correlation in this respect. As an example the authors note that removing every other blade in an impeller does not double the stall propagation speed as the characteristic-time model would suggest. They further note that full-stall propagation speed is insensitive to flow rate, including conditions of flow reversal. This observation would demand that the characteristic time be similarly independent of flow rate including direction of flow. This does not seem to be possible if the characteristic time is related to the boundary layer growth.

Emmons, Pearson, and Grant (1955), responding to the needs of the aircraft industry to minimize the detrimental effects of violent surge in aircraft gas turbines, demonstrated two mechanisms contributing to instabilities in flow machinery. The authors have shown that compressor surge consists of two distinct types of phenomena. First, the whole flow system consisting of intake, compressor and receiving volume may be unstable in the manner of a self-excited Helmholtz resonator. The second mechanism consists of stalling of the flow through blade groups or blade rows in the compressor. This second mechanism, which is usually assumed to be the pulsation excitation for the ultimate Helmholtz resonance, is shown to produce unique effects which may not courie with the resonance phenomena but rather act alone to produce an observable surge.

The authors have noted the variations in published surge descriptions including the fact that reported surge frequencies have sometimes varied with the speed of the compressor and in other reports

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have been shown to be independent of compressor speed. Similarly,
the volume of connected ducting and reservoirs has in some experiments
affected the magnitude of pressure fluctuations in a surging system
but in other studies had no effect. Clearly there could well exist
several modes of instability and, if so, the variety of surge charac-
teristics reported could be due to the occurrence of various combi-
nations of phenomena in any one machine.

The initial experiments performed by the authors using a
centrifugal compressor in a laboratory test stand produced two clearly
defined surge regions characterized by low-frequency pulsing occurring
throughout the system. These two unstable regions were for low
rotational rates separated by a region of stable operation in which
no low-frequency pulsing was observed in the system. In the high flow
rate unstable region the pressure amplitude deviation was mild and
the predominant frequency was 10.5 Hz. The lower flow rate unstable
region was characterized by more violent surge, increasing in magnitude
near zero system discharge and occurring at a dominant frequency of
9.5 Hz. In each case the surge frequency was independent of speed.

The authors' discussion on stall propagation constitutes a
significant contribution to the explanation of surge effects in
rotating machinery. This characteristic is certainly significant
to an understanding of the problems encountered with air-cushion vehicle
lift systems. Whenever the discharge through a compressor is modified
the angle of attack of each compressor blade, either for axial flow or
centrifugal flow machines, is either increased or decreased. The

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Individual blades may therefore approach a stall condition. Emmons,
Pearson, and Grant have shown that as the blade rows approach stall,
the flow may separate on some blades or groups of blades but remain
attached on others. The separated or stalled regions then follow one
another around the circumference of the machine at a frequency which
falls between zero and the rotational frequency of the compressor.
Although this model has been previously described, the authors' explana-
tion of the effect is so logical that it is worth quoting in its entirety.

"A qualitative explanation of this phenomenon is easy to give.
Consider a cascade of airfoils operating on the verge of stall (large
positive incidence angle) and suppose some transient disturbance causes
one vane to stall momentarily (i.e., causes the vane boundary layer to
separate on the convex side); then the flow area between that vane and
the next is restricted momentarily. Since the cascade-exit pressure
remains essentially constant (dependent on the mean turning through
the whole blade row) the local flow must change, and, in fact, the
result is a momentary choking of flow through the single blade passage.
Flow ahead of the blade is diverted toward adjacent passages. The
effect is to increase the angle of attack on the vane next in line in
the direction of tangential component of inlet velocity and to decrease
the angle of attack on the preceding vane. Thus the next vane tends to
stall and the preceding vane tends to become more stable. If the
adversely affected vane does stall, it in turn affects the one next to
it, and in this manner stall spreads along the blade row in the
direction of inlet tangential velocity. But when the stall has

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propagated some distance, the incidence angle on the first vane improves, because of the flow being directed by the choking of succeeding blade passages, so that the first vane unstails, and the unstalling process subsequently moves from blade to blade down the row. One might expect to find stall regions of a characteristic size propagating down the blade row with a velocity which depends upon the rate of formation and recovery of the separation regions.

"The quantitative formulation of the problem involves several steps. By a small disturbance analysis, the condition for the appearance of large-scale disturbances can be obtained, and this is done. Following this, another analysis is required to determine the mean row characteristics observed during stall propagation; such an analysis would treat finite disturbances and would lead to the size of the separation region as well as to the resultant mean characteristics. Such a subsequent analysis is as yet not sufficiently useful to justify the labor involved."

The authors' small disturbance analysis mentioned in the preceding quotations yields a stability criterion for cascades. The flow is shown to be stable or unstable relative to propagation according to whether the gradient of the flow-area ratio with respect to the (tangent of the) inlet angle of attack is less than or greater than, respectively, the ratio of the steady operation flow-area ratio to the (tangent of the) inlet flow angle for steady flow operation. The frequency of propagation of stall is not predicted by the authors' model, as they note in the paragraph quoted from their report.

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The surge model presented by the authors is an energy flux representation for a polytropic process. The final expression obtained for the frequency of pulsation is valid for any two-mass system where the two volumes are connected by the compressor. The final conclusions presented by the authors are however not applicable to air-cushion vehicles since the mass of cushion fluid contained in the receiving volume (i.e., plenum) may vary significantly from its equilibrium value over a full excitation period. The model neglects the third surge excitation mechanism, namely the pumping of the receiving volume which occurs (most probably) at a frequency different from those associated with the instabilities studied by the authors.

Keulegan and Carpenter (1958) have published a solution for the inertial and drag coefficients of two-dimensional flat plates in oscillating (simple sinusoidal) currents. A similar solution for circular cylinders in also included in this paper. The authors present a nondimensional correlation coefficient or period parameter equal to the ratio of the product of the maximum velocity of the sinusoidal current and the period of excitation to the width of the flat plate (or diameter of the cylinder). For the plates, the drag coefficient is shown to be inversely proportional and the inertia coefficient is shown to be directly proportional to this period parameter.

Shimoyama and Ito (1958) have presented a completely experimental study of a pump connected to a large volume receiving tank through a long prismatic-supply tube. The authors reach the following conclusions as a result of their experiments:

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(1) In the surging state the air column in the pipeline vibrates in
     a manner resembling that of a normal mode of free vibration.

(2) Surging amplitude is proportional to the pump revolution rate.

(3) If the length and area of a cross-section of the pipeline are
     fixed, surging tends to build up according to the volume of the
     tank.

(4) Surging is influenced considerably by the conditions under which
     energy is dissipated from the vibrating air column through the
     outlet opening.

(5) The type of surging that usually occurs corresponds to the
     fundamental mode of free vibration. When conditions of the
     pipeline such as the pipeline dimensions and the position of the
     outlet opening have a large damping effect on the vibration of
     this mode, the surging that occurs is of a type corresponding to
     a higher mode.

     Ito (1960) has written independently an encyclopedic work
concerning surging in a blower. His report includes a well-developed
mathematical analysis which is verified through good experimental data.

     The author begins with a survey of past investigations in
the field. An experimental study by Bullock, Wilcox, and Moses (1947)
has indicated that surging is a phenomenon of periodic variation of
pressure and velocity which occurs in the range of discharge in which
the machine has a rising characteristic and that its frequency is the
same throughout the pipeline. A complex dependence was found between
frequency and amplitude of pressure variation and pipeline conditions.

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Ito continues his review of the Japanese literature with a discussion of Fujii's (1948) treatment of the nature of surging as a vibration of a continuous body caused by the rising characteristic of the blower. Shimoyama (1949) is noted for his derivation of a second order ordinary differential equation that provides an approximate description of the phenomenon of surge. His paper also contains a criterion for stability of air flow and a graphical estimation method for the amplitude of surging. Surging of small fan duct systems as investigated by Katto (1960), who has used equivalent lumped models for vibrating air columns, is also noted by the author.

Ito's own experimental research on surging has led him to a number of conclusions:

(1) Stability of a small disturbance in a pipeline with a blower can be determined from the roots of

$$(\bar{b}-4)X^N + \bar{b}X^{N-m'} - G\bar{b}X^{m'} - G(\bar{b}+4) = 0 \tag{3}$$

where $G = \dfrac{\bar{g}-2}{\bar{g}+2}$

$\bar{b}$ is the slope of the blower characteristic curve, $f_B(u)=\bar{a}+\bar{b}u$

$N = m'+n'$

n' is the delivery pipe length

m' is the suction pipe length

$\bar{g}$ is the slope of the valve characteristic curve, $f_v(u)=\bar{a}+\bar{g}u$

u is the velocity of air in the pipeline

If all of the roots of Equation (3) lie within the unit circle on the complex plane, the system can be considered stable.

(2)   If the blower has a rising characteristic ($\bar{b}>0$) at the working
       point, there is the possibility of amplification of a small
       disturbance at that point. If the valve at the end of the pipe
       is fully open or closed, the necessary and sufficient condition
       for stability is $\bar{b}<0$. However, if the blower position coincides
       with a node of any normal mode of vibration of the air column, the
       small disturbance is not depressed even if $\bar{b}<0$.

(3)   If the pipeline cross-section is uniform and the stability
       condition is not satisfied, a disturbance builds up into a fixed,
       sustained vibration that is independent of its initial form. The
       wave form changes along the pipeline, and the final form depends
       on the blower position.

(4)   The wave form of surging is periodic in a strict sense when the
       pipeline cross-section is uniform, but this is not true when
       there is a discontinuous change of cross-section in the pipeline.
       Surging in such a pipeline may, however, be observed as a periodic
       vibration if the dominant mode **is a normal one.**

(5)   The final form of velocity variation at the blower position is
       rectangular only if the valve at the end of the pipe is closed.

(6)   In a pipeline with a high compression ratio blower the frequency
       of surging may be higher or lower than that of the free vibration
       of the standard air column because the air in the suction and
       that in the delivery pipe are in appreciably different states.

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(7) The more rapid the revolution of the blower, the more violent

is the surging and the more important is the role that the blower

characteristic plays in determining the amplitude of surge.

Finally, the author discusses an approximate solution for

surging by formulating the Lagrangian differential equation of motion

which describes the surging phenomenon. The solution builds upon the

wave equations for pressure and velocity of air column vibration. The

resulting model is a two-degree-of-freedom system whose stability

conditions determine the type of surging which will result from a

system with given pump characteristic curves.

Jenny (1961) has extended Bidard's (1946 and 1950) method of

predicting surge from steady flow conditions and the 1950 contribution of

Jenny to include unsteady flow. Both the experimental and the theoretical

results have shown that the surge point is modified under unsteady flow,

with the displacement depending on the frequency and amplitude of the

pressure changes. A drop in compressor efficiency is seen under unsteady

flow, as well. The author has not included the effect of pulse amplitude

on the general compressor characteristics outside the surge region.

Jenny's extended theory is reproduced in the appendix to the present

report.

The optimization of matched compressor systems has been

discussed by Baljé (1962). The use of similarity parameters for

establishing design criteria for optimized systems is demonstrated.
The author shows how the compressor designer may compute the maximum
obtainable efficiencies and optimum design geometry under fixed Mach
and Reynolds number limitations.

One piece of machinery in which surging plays a role detri-
mental to performance is the centrifugal blower of a diesel engine
supercharger. The supercharger operates under conditions in which the
downstream reservoir is subject to flow interruption by the diesel
inlet pistons and high frequency (600 to 4200 Hz) pressure fluctuations.
Of the important work done by Japanese investigators in this area, the
research of Yano (1963) deserves special examination. The author has
reported the results of an experiment consisting of a centrifugal
blower discharging into a straight pipe terminated by a rotary valve.
Yano has applied Jenny's (1950) characteristic method to the system to
compute the temporal behavior of pressure and discharge in the straight
pipe joining the fan to the rotary valve. These results are compared
with measurements obtained by operating the apparatus over a wide range
of excitation frequencies.

From the application of Jenny's characteristic method the
author has drawn several conclusions. First he has found that the
mean pressure (temporal average) at any point in the system is inde-
pendent of frequency. The fluctuating component of pressure (as defined
by a pressure fluctuation coefficient proportional to the difference
between maximum and minimum pressures) is shown to be a function of
excitation frequency and varies with position in the flow system.

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The mean discharge in the system varies with the frequency of excitation. The author further demonstrates an inverse phase relationship between the mean discharge and the pressure fluctuation coefficient.

The experimental results published show some significant deviation from the predicted system response. The mean discharge is generally greater than that predicted by the analytical model and does not usually show the same periodic behavior with increasing frequency. The measured pressure fluctuation coefficient does not show a systematic fluctuation corresponding to the computed cycle rate. In response to these discrepancies the author has included a discussion of the dynamic effects of the compressor and of the rotary valve as distinct from the character of the overall system. The characteristic method assumes that the performance curve of the blower under steady flow can be applied to a system operating under conditions of pulsating flow and assumes similarly that the steady flow pressure-discharge relationship for the rotating valve is also valid for unsteady flow conditions. These assumptions were examined by the author by computing the appropriate excitation frequencies or reduced frequencies for each of the mechanisms. By this technique the author was able to conclude that for his experimental system the rotary valve had qualitatively more distinctive unsteady characteristics than the compressor. The performance of the blower under unsteady flow conditions was therefore felt to resemble its steady flow performance much more than expected.

An experimental investigation of the flow characteristics of a centrifugal compressor under unsteady flow conditions has been

performed by Benson and Whitfield (1965). At low flow rates the operating range of the compressor is limited by surge and at high flow rates by choking. The authors tests have shown that under unsteady flow conditions the surge point is displaced to a point of greater mass flow, thereby reducing the flow range of the compressor. The magnitude of this displacement is dependent upon the frequency and amplitude of the pressure pulses. It is seen that pressure pulsations can be transmitted through the compressor and that surge can be induced under certain conditions when normal steady flow matching would not indicate surge.

Overall compressor efficiency is seen to be reduced under all test conditions, with the decrease in efficiency apparently not monotonically dependent on the pulse frequency. In contrast, the authors cite Jenny's (1961) experiments which indicate that a drop in efficiency is related only to the pulse frequency. Jenny's results have shown that the higher the frequency, the smaller the decrease in efficiency.

The authors show that the flow characteristics of the compressor are displaced under all conditions of pulsating flow and that the deviation from steady flow characteristics is greatest at low frequencies and with the largest fluctuations in rotary valve area. Their study has not established whether the results are due to characteristics of the delivery system alone or to the combined characteristics of the delivery system and the compressor.

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Finally, the authors present an outline of the analysis of surging developed by Bidard (1946, 1950) and extended by Jenny (1961). Their summary is included in the appendix to this report.

Myles (1965) has presented a method by which the cascade behavior of aerofoils is superimposed on a centrifugal flow field. He has shown that the use of two-dimensional aerofoil cascade performance data in the design of mixed flow fans allows the overall fan performance to be estimated within reasonable limits of tolerance. He has demonstrated in this way that the boundary-layer behavior does not appear to have a great influence on the lift behavior of blades at the design condition. No comparable conclusion is drawn for off-design performance, however, especially for near-stall conditions.

This paper is part of a series of reports by Myles and Watson (1964) and Myles, Bain, and Buxton (1965) dealing with computer design of fans and compressors. The application of this work was summarized by Hesselgreaves and Jones (1970) as briefly discussed in a subsequent paragraph.

Comparison of the static characteristics of ACV lift system fans has been improved by Burgess (1965) who has devised a new method for reducing data on hovercraft fan performance. In order to eliminate the dependence of previously published performance curves on model weight, Burgess demonstrates that presentation of volume flow data in the form of the ratio of fan discharge to the square root of the craft weight yields a relationship uniquely related to hover height (independent of weight). The author further proposes that for practical purposes the fan speed be normalized by the square root of the craft

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weight and that the required power be normalized by the craft weight raised to the three-halves power. This presentation also yields relationships which are uniquely related to hover height over a range of craft weights. These results are derived for rigid seal ACV's; however, the author concludes that for flexible seal hovercraft the extent to which the weight-normalized relationships are borne out in practice is a measure of the extent to which the flexible seals deform under the differing inflation pressures which result from various craft weights.

Ohashi (1967) has performed an experimental study of the pressure characteristics of a centrifugal pump when the flow rate was forced to vary sinusoidally around an average discharge. The experiment showed that the amplitude of pressure rate fluctuation decreased and the phase lag of pressure rate fluctuation increased when the frequency of fluctuation exceeded a frequency given by

$$f = C_f \frac{N_R \phi}{\cos \lambda_R} \quad n \qquad (4)$$

where f is the frequency of forced fluctuation (H_3)

N_R is the number of biades

ϕ is the flow coefficient (rps)

λ_R is the stagger angle of the cascade (55°)

n is the rotational speed of the impeller (rps)

C_f is the frequency coefficient

Ohashi's theory gives $C_f = 0.3$, and the experimental results yield $C_f = 0.1$.

Experimental data indicate that the amplitude of the change

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of pressure rise decreases with the increase of reduced frequency but at a faster rate than the Ohashi theory predicts. The phase delay of the change of pressure rate relative to the quasi-static case increases with the increase of reduced frequency, but at a rate higher than that predicted by the theory.

Parker (1967) has analyzed the resonance effects due to wake shedding from the blades of compressors through an examination of the sound pressure levels excited at a series of acoustic resonances. The author has found the tendency to shed periodic wakes to be greatest for the lowest blade loadings and to disappear prior to the incidence of stall. The author observes that the resonance mode (integral number of waves excited) is not necessarily a function of the number of compressor blades and that acoustic wave lengths correspond to propagation velocities less than sonic for an unexplained reason.

Parker (1968) also has developed an irrotational flow model of compressor cascades in relative motion. Besides irrotationality, his method is limited to the flow of incompressible fluids. A finite difference formulation is presented to evaluate the stream function through relaxation. The model therefore provides a means of estimating the pressure fluctuations which are responsible for the generation of unsteady forces on blades with small blade row spacings. The calculations seem to be adequate for low and medium speed axial flow machines which are not subject to conditions of stall. The author indicates that for typical axial flow units, the potential flow interaction may be much greater than blade wake effects except when the blade row spacings are

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large. The mechanism of potential flow interaction operates through

the prevention of accelerations in the direction normal to the guide

vanes of the stator. This produces large unsteady pressure differences

across the vanes resulting in large vane forces. The author's method

may also be used to calculate the flow through single cascades or

radial flow machines using either a stationary frame of reference or

one moving with the rotor blades.

Helicopter rotor foils experience rates of increase in angle

of attack which are often nearly constant over a significant portion

of the blade cycles. Ham and Garelick (1968) performed two-dimensional

airfoil experiments with a linearly increasing angle of attack in order

to study the dynamic stall effects of a helicopter rotor. They have

concluded that the rate of change of angle of attack is the dominant

factor in determining the very large peak lift and moment acting on

the airfoil during the process of stall. They further conclude that

the location of the pitching axis has a relatively small effect on the

peak loading but a substantial effect on the angle of attack at which

dynamic stall occurs. This suggests that the loading is relatively

independent of the dynamic stall angle.

The authors have measured the chordwise distribution of

pressure and have concluded that "dynamic stall occurs at angles of

attack ranging from 15 to 20 degrees, and is indicated by the decreas-

ing of leading-edge suction when dynamic stall occurs. A pressure

disturbance moving aft from the leading edge simultaneously increases

the suction in the midchord region. Still later, the leading-edge

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suction further decreases, while the pressure disturbance moves still
further aft, and is of relatively large magnitude. The character of the
disturbance suggests that it consists of free vorticity introduced
into the blade flow field from the neighborhood of the blade leading
edge as the blade loses bound vorticity during the dynamic stall
process. The results indicate that the dynamic stall phenomenon has
far different characteristics than those associated with the classic
trailing edge separation during static stall of a blunt airfoil."

The authors have also found that a high rate of change of
angle of attack delays (in time) the occurrence of stall. Evidently
the sustained upper surface suction associated with the chordwise
passage of the vorticity shed during the dynamic stall process contri-
butes to high sustained lift values.

There is a specific heaving velocity of the airfoil which
produces a maximum stall angle of attack for any pitching velocity.
The critical heaving velocity is negative (positive up) and the
maximum stall angle increases with increasing (negative) heave velocity.
It is found that there is a decreasing sensitivity of the maximum stall
angle to heaving velocity with increasing pitching velocity. It is
also concluded that the onset of dynamic stall occurs when the air
foil is pitching about the leading edge.

Finally, the authors have presented their experimental data
for the peak values of lift and moment in terms of the pitching velocity
parameter $C\dot{\alpha}^{(S)}/V$ where C is the chord length, $\dot{\alpha}^{(S)}$ is the rate of change
of angle of attack of the airfoil evaluated at the instant that dynamic

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stall occurs, defined as the instant when suction at the 10 percent chord
station no longer increases with increasing airfoil angle of attack,
and V is the free stream velocity. Their results are reproduced as
Figure 4. "Both the experimental and the theoretical data indicate
that the maximum vortex-induced loading due to dynamic stall is
relatively insensitive to pitch axis location, i.e., quarter-chord
heaving velocity, for a given value of velocity parameter. This
result is of particular importance in the helicopter application,
since the instantaneous angle of attack of a helicopter blade element
is determined b the instantaneous blade element pitch angle, heaving
velocity, and wake-induced downwash. The significant effect of rela-
tively small values of heaving and/or downwash velocity on the dynamic
stall angle of attack, therefore, is of little consequence in deter-
mining the loading: the time gradient of angle of attack, rather than
the magnitude of the angle of attack, is the primary factor, at least
for the small values of quarter-chord heaving velocity corresponding
to the range of pitching axis locations considered here."

The authors conclude that "the experimental results of the
investigation indicate that at moderate to high pitching rates the
aerodynamic loading on a two-dimensional wing during large amplitude
pitching motion is dominated by the influence of intense vorticity
shed from the vicinity of the wing leading edge following the occurrence
of dynamic stall at an angle of attack substantially greater than the
static stall angle. In the case of oscillatory pitching motion, this
vortex-induced aerodynamic loading generates adversely phased pitching

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moments that sustain the motion at certain values of reduced frequency
and mean pitch angle, and the phenomenon known as stall flutter results.
In the present case of arbitrary motion to high angles of attack, the
vortex-induced aerodynamic loading generates peak values of dynamic
lift and moment on the airfoil that are substantially greater than the
maximum static values achieved on the same airfoil." Although the authors'
research is not presently applicable to the ACV/SES dynamics problem it
should find immediate use with the advent of the variable geometry lift
fan in large scale SES applications.

Fujii (1968) has formalized a method for calculating the
aerodynamic characteristics of a cascade of airfoils which is subjected
to inlet distortions. The author's solution is steady, contains no time-
dependent terms, but is otherwise exact in that the exact cascade and foil
geometry is used in lieu of thin disc approximations.

In 1968 N.D. Ham formulated a vortex theory to represent the
process of dynamic stall through the shedding of vorticity from the lead-
ing edge of an airfoil. Two vortices are assumed to constitute the dynamic
lift model. Bound vorticity lost from the leading edge of a foil is assumed
to take the form of free vortex elements which move with the velocity of
the local flow and which have sufficient strength to maintain a stagnation
point at the leading edge of the foil. A second trailing edge vortex is
assumed to exist with a conventional (classical unsteady airfoil theory)
trailing edge stagnation point. Trailing edge vortices are shed under
the condition that the stagnation point is preserved. The conditions of
leading- and trailing-edge stagnation points and Kelvin's theorem of

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constancy are sufficient to determine the strengths of the two vortices, the blade-bound circulation, and the entire flow field at any time through classical potential flow theory. The resultant model is two-dimensional, and the author restricts consideration to flat plate (zero camber) foils of negligible thickness.

Through this model and the interpretation of experimental results the author has concluded that the aerodynamic loading on a two-dimensional plate subjected to large amplitude pitching motion is dominated by the influence of the intense vorticity shed from the leading edge following the occurrence of dynamic stall at an angle of attack substantially greater than the static stall angle. Out-of-phase pitching moments are produced by the vortex-induced loading. The phase relationship leads to stall-flutter conditions at certain values of excitation frequency and mean pitch angle.

The author's theoretical formulation is demonstrated to predict the experimental magnitude of peak oscillatory moment with fair accuracy and provides a reasonable prediction of the peak experimental vortex-induced lift (Figure 5). The author has noted an important conclusion resulting from the loss of leading-edge suction during the vortex shedding. Since the vortex-induced pressure drag is equal to the streamwise component of the resultant aerodynamic force normal to the airfoil, at large angles of attack the vortex-induced drag will exhibit large peak values. Finally, the author's theory has demonstrated accuracy in predicting the temporal and spatial unsteady pressure (Figure 6).

The author's model does not consider viscous-flow effects. A complete theoretical description of the shedding of leading-edge vorticity during dynamic stall is therefore beyond the capability of the author's present formulation. For this model, the initiation of both leading- and trailing-edge vortex shedding has to be determined empirically.

West (1968) has given a presentation of theoretical and experimental results for the airflow and power requirements of a compressible hover-craft jet. His report is concerned with the compressibility in the air flow under ground effect machines and not with fan dynamics per se.

An experimental and theoretical investigation of the pressure fluctuations produced by interaction between blade rows in axial flow compressors has been summarized by Parker (1969). The potential flow interaction effects (theoretical results) discussed were presented by Parker in two earlier papers (1967 and 1968). The present paper describes a series of experiments performed using axial flow devices for the purpose of establishing the validity of the analytic model. The experimental results show that in general the potential flow interaction between blade rows is a significant source of generation of fluctuating pressure in the machinery. Parker has found that the passing of blades behind flow obstructions such as guide vanes produces rather large pressure fluctuation on the surfaces of the guide vanes. The relative amplitude of the blade-excited pressure fluctuation is adequately predicted for the various geometries tested but the absolute magnitude of the pressure pulses is not accurately predicted by the author's analysis. Parker proposes the possibility of unsteady flow boundary layer development in the fan as an explanation for some of the anomalous results

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observed. For example, the analytic model predicts larger pressure amplitudes on the downstream (away from the approaching rotor blades) side of the guide vanes than on the upstream side. Unfortunately the experimental results indicate just the opposite behavior.

Hysteresis type behavior in the dynamic behavior of an edge-jet hovercraft has been described by Davies and Poland (1970). The mechanism for hysteresis in edge-jet craft is not related to the fan-cushion system noted heretofore but rather to nonlinear Coanda behavior in the flow of cushion fluid under the edge-jet contour. Two states of flow have been identified by Davies and Poland and have been analyzed separately. In the first case, termed "underfed", the edge-jet flow is augmented by a jet of cushion fluid produced by the decrease in the cushion volume as the craft moves downward relative to a fixed ground surface. The second flow state, denoted as "overfed", occurs when the edge-jet craft is moving upward. The increase in cushion volume requires that the edge-jet split so that a stagnation point occurs somewhere near the cushion boundary. A portion of the jet is therefore deflected into the cushion. Davies and Poland have indicated a procedure for joining the two flow states through a switching condition in which the switch occurs when the thickness of the cushion jet in the underfed state or of the inward jet in the overfed state becomes zero. The result is a discontinuous jump from the overfed to underfed stages or vice-versa.

In a sequence of numerical experiments the authors have

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demonstrated that the nonlinear behavior discussed results in a dependence of the dynamic equilibrium position on the input frequency. The authors have found the hover height to be a linearly decreasing function of oscillation frequency within the range of frequencies examined. The nonlinear character of the system is also demonstrated in the locus of points relating cushion pressure to hoverheight clearance which presents itself as a closed loop for the two phases of motion or states of edge-jet flow. The authors conclude that during the overfed phase there is a greater loss of pressure than there is a gain during the underfed phase. They observe that the ratio of gain to loss of cushion pressure in one complete cycle of the steady state oscillation is approximately three.

The nonlinear mechanism described by Davies and Poland would naturally couple with other nonlinear flow phenomena, compounding the actual dynamic behavior of edge-jet type hovercraft. The present description would appear to be valid for these vehicles in which other nonlinear characteristics are predominant at excitation frequencies outside of the range in which the edge-jet switching model shows significant response. The edge-jet model does however promote the consideration of nonlinear models. The authors sound a discouraging note for the use of linear mass-spring-dashpot systems (e.g., Duke and Hargreaves (1962)) to describe the motions of edge-jet hovercraft.

The design of air-cushion-supported vehicle lift fans by numerical methods may prove to provide optimized systems. Hesselgreaves and Jones (1970) have provided the outline of a machine-designed axial

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fan and have presented the results of a series of physical experiments
verifying the flow characteristics of the numerically designed unit.
The future inclusion of unsteady and dynamic effects for an entire
lift fan system in an optimum design will certainly require machine
optimization of a form presumably similar to that employed by the authors.

The results of some rather extensive research on the unsteady
performance of a diesel engine supercharger are reported by Yano and
Nagata (1971). This work builds upon and extends the results of similar
research reported by Yano (1963). In the present paper the authors
introduce and clarify several significant aspects of the problem. First,
they clearly show how the "apparent" blower characteristics compare
with the steady flow characteristics of the blower. In many design
applications which are not concerned with detailed dynamic behavior,
the average level of performance is sufficient to display the effects
of surge phenomena. The "apparent" fan curve which represents the
relationship between the time-average values of the mass flow and
the pressure ratio therefore indicates the mean response of the system
relative to the steady flow or zero frequency performance. The apparent
characteristic curve will generally follow the steady flow fan curve
along **the** negative slope portion of the curve. In positive slope
regions, however, significant deviations can be expected, usually
manifested as a decrease in pressure differential for any particular
discharge.

Yano and Nagata also show the difference between forced
oscillations occurring at a forced frequency f_R and surging oscillations

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occurring at a separate (sometimes equal) surging frequency f_S. The relationship between these two frequencies is linear, involving the integral number of repeated acoustic waves n which can exist in the length of duct work separating the compressor from the source of excitation. The relationship presented by the authors is $f_S = f_R/n$.

The authors include a discussion of the effect of moving pressure waves in the characteristic method employed in their numerical analysis. The basic conclusion reached is that the actual pressure history may be neglected when the air column length is a small percentage of the volume of air contained in the system. For long, slender duct work systems the effects of the pressure wave are too great to be neglected.

A series of experiments was performed in which a resistance (flow blockage) plate was inserted into the system to change the operating point of the blower. This allowed the authors to exert a great deal of control over the incidence of surging in their forced excitation studies. Through this means the proportional relationship between forcing and surging frequencies was firmly verified. In presenting their final results showing relationships between pressure coefficient and excitation frequency and flow coefficient and excitation frequency the authors were able to conclude that without surging, both pressure and discharge decrease rapidly with increasing excitation frequency. With the inception of surging, however, the pressure coefficient decreases only slightly with increasing frequency while the discharge coefficient exhibits the opposite behavior, increasing mildly with frequency.

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McCroskey and Fisher (1972) have postulated a sequence of
events in the dynamic stall process that support the three basic
contentions of Ham and Garelick (1968) regarding the mechanism of
retreating blade stall for temporarily changing angle of attack. The
three phases identified by Ham and Garelick with a given rate of change
of angle of attack are (1) the leading edge suction and maximum normal
force increase above static levels, (2) the shedding of a leading edge
vortex and the resulting leading edge suction loss, and (3) complete upper
surface separation. This list has been expanded by the authors into
a nine event process. The sequence of events for the retreating blade
stall region includes:

(1) Onset of separation-like disturbances within the boundary layer
    and a slight shift to the center of pressure.

(2) Beginning of a rapid rearward shift of the center of pressure and
    increase in the blade element moment coefficient.

(3) Collapse of the leading edge suction peak.

(4) Achievement of maximum blade element normal force coefficient.

(5) Achievement of maximum blade element moment coefficient.

(6) Completion of the stalling process.

(7) Reestablishment of leading edge suction.

(8) Stabilization of the center of pressure.

(9) Reestablishment of normal boundary layer characteristics.

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University has recently provided itself with a capability for detailed
research on the response of axial flow turbomachinery blade elements
and/or blade rows to unsteady flows. Bruce (1972) has described the
axial flow research fan and the instrumentation available for experi-
mental dynamic response of blades. The dynamic excitation is provided
through disturbance-generating grids with circumferential variations
in blockage rather than by a homogeneous pulsation of the fan flow.

The unsteady stall characteristics of two-dimensional foils in
an incompressible fluid have been examined by Crimi and Reeves (1972).
They report the development of a method which utilizes mathematical
representations of several flow elements involved in unsteady airfoil
stall. The flow elements included in the model differ for various stall
classifications. The three stall classifications considered by the
authors are (1) leading edge stall, (2) trailing edge stall, where the
flow separation streamline springs from the upper surface of the foil
at some point between the leading and trailing edges of the foil, and
(3) thin-airfoil stall characterized by a laminar bubble springing
from the leading edge of the foil at a small angle and reattaching to
the foil at some downstream point. This type of stall occurs only at
low Reynolds numbers on thin foils.

The primary flow elements appropriate to leading-edge stall
conditions are (1) a laminar boundary layer to the point of separation,
(2) a laminar constant pressure shear layer to the point of transition,
(3) a turbulent constant pressure shear layer, (4) a turbulent pressure-

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recovery region and (5) a potential flow region over the foil and external to the viscous mixing region and vortical wake. For trailing-edge stall the model must include the elements listed above plus (1b) a leading edge bubble if laminar separation occurs prior to separation and (1c) a turbulent boundary layer if transition precedes separation. For conditions under which the flow is everywhere attached or reattached to the foil, the model consists of (1) a laminar boundary layer extending over the leading edge, (2) a leading edge separation bubble, (3) a turbulent boundary layer from the reattachment point of the leading edge bubble or the transition point to the trailing edge and finally (4) a potential flow model for the flow over the airfoil, including the effects of a vortical wake generated by the unsteady circulation about the foil.

To facilitate numerical solution each flow element was considered qualitatively as simply as possible. Linearized potential flow analysis was applied. Quasi-steady flow was assumed for the viscous mixing regions and the leading edge bubble. Boundary layer analysis was performed, however, via a complete second-order unsteady finite-difference method. Finally, a numerical iteration technique was employed to relate the various interacting flow elements.

The authors demonstrate the method through comparisons of computed and measured loads on a pitching airfoil. The generally good qualitative agreement is taken as a sign that the method is capable of reproducing the essential features of dynamic stall. Most significantly, the authors conclude that their results indicate that the dynamic over-

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shoot of the normal force on an oscillating foil is caused by a re-
distribution of pressure associated with the pitch rate as well as
loading induced on the aft portion of the airfoil by the presence of
a dead air region on the forward end of the foil.

Johnson and Ham (1972) suggest that the laminar separation
bubble is responsible for dynamic stall. The proposed model involves
a bubble which is stretched by the unsteady transition process and
subsequently bursts as the reattachment point enters a region of high
adverse pressure gradient. Isogai (1970) has indicated in this con-
sideration that bubbles as long as 0.15c (where c is the chord length)
may be possible.

Philippe and Sagner (1972) review experimental and theoretical
methods of computing and measuring unsteady aerodynamic forces on
oscillating airfoils developed in France, including unsteady stall
effects. One feature of the authors' experimental method is the use
of a force balance within a section of the oscillating airfoil. This
approach has the advantage over the pressure measurement technique of
permitting the measurement of unsteady drag forces on the airfoil in
addition to lift and moment. The drag forces are shown to up to one-
third of the magnitude of the lift forces during oscillations through
stall. The highest amplitude of oscillation tested by the authors was
$\pm 6^{\circ}$, so the experimental data is applicable to the study of stall
flutter oscillations at blade torsional frequencies but not to the
overall blade dynamic stall onset resulting from variations in the
angle of attack at rotor rotational frequencies.

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An experimental and theoretical study has been performed by Thomas (1972) to develop a method for predicting the stiffness and damping parameters of a hovercraft which are attributable to changes in cushion pressure during forced excitation of the cushion. The experimental study was of two parts. The first consisted of a fixed model hovercraft suspended over a "pitching-heaving" table with a rigid surface capable of performing sinusoidal motions. The second series of experiments was carried out with the hovercraft free to pitch and heave over the "pitching-heaving" table. All experimental work was therefore restricted to zero speed.

The theoretical model discussed by Thomas is an attempted extension of Wheatley's (1969) small amplitude theory to cover large excursions in the relative motion between the hovercraft and the rigid surface. The attempt was unsuccessful, and the author has therefore limited the analytic model to an examination of the detailed effects of skirt motion.

Thomas concludes from the appearance of the cushion pressure hysteresis loops as a function of relative craft-table displacement that the dynamic behavior is produced by a rather complicated sequence of changes in the physical significance of cushion stiffness and damping as a function of time (or relative displacement) in an excitation cycle. The author proposes that:

"To obtain the final proposed solution, motion can therefore be considered in four parts:

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a. forcing surface moving upward from its equilibrium posi-
tion and the model fixed thereby compressing the cushion.

b. moving downwards to the equilibrium position,

c. moving downwards from the equilibrium position to the
lowest position,

d. moving upwards from its lowest to the equilibrium position.

a. Due to the action of forcing, it has been suggested
earlier that stiffness in this section of motion can be considered as
linear, the stiffness of the cushion being given by the slope of the
static stiffness curve at the equilibrium position. Superimposed on
this stiffness is the contribution due to damping....

b. The motion downward from the maximum upward position of
the forcing surface again follows the upward motion as regards the
stiffness of the cushion, but with damping neglected as a first
approximation, (peripheral jet theory suggesting a ratio of the order
of 3:1 as mentioned previously) as values of damping at low frequencies
and amplitudes were relatively small.

c. Damping in this segment of motion is again neglected for
reasons discussed previously, and due to the air gap, stiffness is
given by the corresponding part of the static stiffness curve.

d. The return to the equilibrium position to complete the
cycle is considered as the corresponding section of the static stiff-
ness curve as c) above, but with Wheatley's value of damping super-
imposed to define the motion.

"Summarising the above, motion upwards of the forcing surface may be considered as a 'dynamic stiffness' as discussed, with damping superimposed, the value of that damping given by Wheatley's theory. Motion downwards of the surface, again considering a fixed model, is defined by the 'dynamic stiffness' term only as a first approximation. It is suggested that this approach to the problem of predicting pressure changes within the air cushion may be valid over a reasonable range of forcing frequencies and amplitudes."

Unfortunately, this model, while seeming to yield reasonable agreement between measured and predicted pressure variations, does not account for the effects of the dynamic response of the fan system or cushion seal system. The deviation from the experimental results increases dramatically with both increasing frequency and increasing amplitude.

Wheatley's small amplitude theory is also poor in predicting cushion pressure. This model predicts only qualitatively the hysteresis in measured pressure. The substitution of a more accurate (quadratic) fan characteristic into Wheatley's model (in place of the linear pressure-discharge relationship) does improve the analytic predictions, but this is only a result of the rather large excitation amplitudes used in the experimental program. Other investigators have demonstrated hysteresis behavior even for low amplitude excitation.

Although the validity of the author's analytic formulation

may be subject to question, especially the author's introduction of "dynamic stiffness", certain of the published conclusions deserve direct quotation.

"Over the range of amplitudes and forcing frequencies tested, fan speed remained sensibly constant throughout. This confirms reports of little variation in fan speed on full-scale craft.

"During forcing in heave, pitch, and heave and pitch combinations, the pressure distribution over the base of the craft was sensibly constant at any instant in time (with perhaps some slight local variation) i.e. cushion pressure or plenum pressure is displacement (i.e. time) dependent only. The stiffness of the air cushion in pitch was therefore not provided by a pressure gradient in the air cushion, but was found to be attributable solely to the shift of the centre of pressure of the cushion, thus providing a restoring moment. This demonstrates the designed inherent stability of the H.D.L.-type skirt. The pitch stiffness of the model hovercraft was found to be 1.0% C.P. shift/degree over the range $+2.5^{0}$ to -2.5^{0}, a result which is in fair agreement with that of H.D.2 full-scale.

"The behaviour of the skirt of a hovercraft may be a crucial factor in the overall behaviour of the craft and affect both the stiffness and the damping of a craft at a given instant. The model skirt, in this case, followed the motion of the forcing surface whilst the cushion was compressed, but as the craft and forcing surface moved

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apart the skirt tended to remain in its compressed state for an

appreciable time before finally 'cracking' back into its extended

position. This observation led to the concept of a 'dynamic stiffness'

of the air cushion in heave, and also that damping may be considered

to be present only when the cushion is being compressed, the remainder

of the motion being dominated by the stiffness term. The use of the

'dynamic' stiffness concept eliminates the necessity to measure h_e

[mean height of cushion exit area in static conditions], as the stiff-

ness is measured directly from the results of a 'zero' speed forcing

test as discussed in this thesis.

"The forcing of the air cushion through large amplitudes

giving correspondingly large changes in cushion pressure and air

volume flow invalidates the use of a linear relationship for the fan

characteristic, and the substitution of a quadratic expression was felt

to be more accurate. However, at the higher frequencies of forcing,

the prediction of air volume flow is difficult to obtain as the pressure-

volume flow relationship of the static fan characteristic is not followed.

It is felt that some form of dynamic characteristic as suggested by

Benson and Whitfield (1966) may hold, but further work is needed in this

area both experimentally and theoretically. Nevertheless, the use of a

quadratic expression, despite the limitations, ...has given rise to

theoretical predictions which correlate well with experimental results

over the range of amplitude and frequencies tested.

"The theory of Wheatley applied only to heave motions and has

been extended on this basis. Further work is required on the extended

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theory for the accurate prediction of volume flow changes. The
extension of the theory to cover pitch, and combined heave and pitch
motions has been suggested, and a tentative analysis shows little
coupling between pitch and heave thus simplifying the problem. It is
felt that further theoretical assessments could be made of the pitch
cases in order that craft behaviour can be completely understood."

A report by Ward and Young (1972) presents a survey of
research activity in rotor unsteady aerodynamics, including the NASA
Langley Research Center program. The authors describe the unsteady
aerodynamic environment of the rotor and discuss trends in the state
of the art. They indicate that the current tendency is toward complex
mathematical computer models of rotor aerodynamics. They discuss
experimental and theoretical research on rotor maneuver loads, level
flight and maneuvering wake prediction, tip-vortex flow, blade-vortex
interaction, dynamic stall, transient compressibility effects, and
variable geometry rotor research aimed at alleviating blade-vortex
interaction effects.

Aerojet General Corporation (1973) has reported on the lift-
system characteristics of the Aerojet design for a 100 ton surface
effect ship (SES-100A). The report includes a discussion of the active
heave attenuation system (using cushion vent valves) designed to reduce
heave accelerations during craft operation in rough water. The system
operates by venting cushion air to the atmosphere in response to ship
vertical acceleration. The cushion pressure is thereby reduced and the
heave acceleration "minimized." During this operation "the vent valve

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bleed-off may be as much as 75% of the fan air flow which during venting must ideally be replenished by the fans." The system therefore requires that the lift system discharge capability be dramatically overdesigned. Operation of this system also presupposes that the fan flow rate and pressure head are noticeably unsteady. The effects of unsteady operation have not been discussed by Aerojet in this report. The authors have limited the scope of the presentation to a discussion of the steady or static flow requirements of the system.

Bass and Johnson (1973) have written an interesting report concerned with the dynamic response of bag-type seals subject to flutter and other unsteady flow effects. These effects imply non-linear behavior in the air-cushion vehicle cushion but are not directly concerned with fan dynamics. The authors' work should be further examined in an analysis of cushion dynamics.

In a technical evaluation report on aerodynamics of rotary wings Ham (1973) has noted deficiencies in the knowledge of some fun-damental aspects of the physics of rotor flows. The author states that detailed descriptions and models are needed for a number of physical processes, among which are the following: (1) the formation of a blade tip vortex, (2) the structure of a blade tip vortex, (3) the rolling up of the blade trailing wake, (4) the interaction of a tip vortex with a blade, (5) the onset of dynamic stall on a blade, (6) the loading on a blade due to dynamic stall, and (7) the effects of compressibility on the previously-listed processes.

The analytic model employed by McCroskey and Philippe (1975) is described by McCroskey (1973). In this presentation the author

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furnishes several simple thin airfoil formulas to describe the in-
viscid, incompressible flowfield about an airfoil operated in an un-
steady fashion for various degrees of thickness and camber. The
potential flow theory predicts the magnitude and phase of leading
edge pressure gradients (dynamic delay in laminar separation) for an
oscillating foil but does not predict dynamic stall characteristics.

The author's theoretical method consists of three distinct
parts. The first is the linearization of the boundary condition on
the airfoil and in the wake, thereby permitting the velocity potential
to be separated into components of thickness, camber and angle of
attack. Second, the potential flow solution is formalized and Allen's
method for steady airfoils is employed to improve solution accuracy.
Finally the unsteady wake effects are described via Theodorsen's method
for unsteady flat plates.

Although the author's theory is limited to inviscid flow, the
combined theoretical and experimental results indicate that unsteady viscous
effects on oscillating airfoils are less important than the unsteady
potential flow effects. This opinion is however based upon the assump-
tion that the boundary layer does not interact significantly with the
main flow. The author points out that this assumption is valid for
dynamic stall. For that phenomenon, the inviscid theory predicts the
onset of boundary layer separation but does not predict the delay in
separation (dynamic delay above the static stall angle). Further, it
is noted that the inviscid model underestimates the angles of attack
where the pitching moment and lift first exhibit indications of stall.

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Martin, Empey, McCroskey and Caradonna (1974) have reported
the results of an experimental study into the time-dependent behavior
of a single oscillating two-dimensional NACA 0012 airfoil. The authors'
experimental observations resulted in the detection of a short laminar
separation bubble preceding the stall condition. The authors were
not however able to verify the presumption of Johnson and Ham (1972)
that dynamic stall is caused by the bursting of the separation bubble.
From smoke-flow movies Martin, et al. have observed the formation of
a conspicuous laminar separation bubble during the downstroke phase of
the airfoil oscillation. This bubble was seen to move slowly downstream
as the angle of attack was increased and, at its maximum, extended back
along the foil to a length of 0.2c. There was no indication of a
formation of a similar bubble during the upstroke of the oscillation
(decreasing angle of attack).

The results presented by the authors are conveniently compared
with the model-rotor data of McCroskey and Fisher (1972) and the data
of Ham and Garelick (1968) for linear variations in angle of attack.
Although the three studies were quite different they tend to indicate
two similar results. First there is a stall delay as the angle of
attack is increased (compared with the static stall point) and second,
in each case, there is a clear indication of vortex shedding with
respect to vortex formation. Martin, et al. have concluded that it is
normal for more than one large vortex to be shed as the angle of attack
is increased. For higher oscillation frequencies there does not appear

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to be time for the formation of two vortices and only one large vortex is shed during an upswing phase in the oscillation. A leading-edge vortex is always shed when the angle of attack reaches its maximum, and the authors have noted that this process appears to be different from that which occurs on the upswing.

The major finding with respect to stall initiation is that the angle of attack for stall initiation decreases with increasing Reynolds number (for Re = 3×10^6). This is not in accord, unfortunately, with the results of Ham and Garelick (1968) or McCroskey and Fisher (1972) whose experiments were performed at lower Reynolds numbers (Re = 3×10^5). The authors conclude therefore that the stall processes may differ for these two different flow regimes. With respect to frequency effects, the authors have shown that the maximum normal force on the foil, the maximum pitching coefficient, and the angle at which lift stall occurs all increase as the frequency of oscillation is increased. (The experiments were performed at reduced-frequencies k = wc/2V of 0,0.05, and 0.1.)

Maximum lift on the foil is found to increase with Reynolds number. Further, it is observed that the angle of attack at which minimum pressure was measured was not always the same as the angle at which the maximum leading edge velocity occurred. The incidence of maximum velocity usually occurred first, indicating that separation was occurring directly at the leading edge during the initiation of stall. The authors therefore state that the most appropriate criterion for indicating the initiation of the stall process is the peaking of the leading-edge velocity.

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A "Fan Evaluation Rig" has been constructed recently at the David W.
Taylor Naval Ship Research and Development Center. The facility consists of
a 640 cubic foot plenum connected to the atmospheric test section of a
subsonic wind tunnel by the fan under consideration. The plenum is
vented to the atmosphere through a 30-inch diameter variable frequency
butterfly exhaust valve which may be operated reliably at frequencies
between zero and 5 Hz. The plenum pressure or back pressure may also
be augmented through a supplementary air supply. Durkin and Langhi
(1974) presented the results of their first experimental evaluation of
the dynamic response of the fan-plenum system at the International
Symposium on Air Cushion Vehicle Technology in 1974. The authors'
objective was to measure the steady-state performance characteristics
of an Aerojet Liquid Rocket Company centrifugal flow lift fan. Their
second objective was to determine the fan characteristics in a dynamic
environment obtained by sinusoidally varying the area of the exhaust
duct from the test plenum at frequencies from 0.5 through 6.0 Hz. The
overall objective of the investigation was to evaluate techniques to
be used for evaluation of fans under conditions of unsteady flow.

The authors have published a set of pressure-discharge hysteresis
curves representing the dynamic behavior of the fan-plenum system for
various excitation frequencies, an example of which is given as Figure 1.
These curves show that for a fixed fan speed the magnitude of pressure
and discharge variations due to sinusoidal excitation decrease with
increasing frequency in the frequency range of zero to 5 Hz. The results
also show that the average discharge increases with increasing frequency;

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however, the average pressure rise across the fan decreases as the excitation frequency is increased. If the published figures are interpreted correctly, the dynamic data also demonstrates that the pressure-discharge hysteresis curves may fall entirely below the steady state fan map.

The dynamic data is limited by the fact that reverse velocities at the fan could not be measured and therefore the dynamic hysteresis loops which should obviously extend over into the domain of negative flow coefficients are truncated at the origin. The authors have mistakenly shown these open curves to be closed along the ordinate of the head-flow coefficient graphs.

The authors conclude by noting that "the measured dynamic performance data show that during cyclic operation energy is periodically stored and released in the (rotor-volute-plenum) system." They state that their "preliminary data analysis has shown that the fluid inertance in the volute and the compliance of the plenum are probably responsible for the storage. However, the preliminary data do not permit the quantisizing the combined effect of these elements."

McCroskey and Philippe (1975) have investigated experimentally and numerically the unsteady incompressible laminar and turbulent flow over flat plates and airfoils. In general the authors have observed that unsteady effects on laminar boundary layers decrease with increasing longitudinal pressure gradients. Turbulent separation on the airfoils tested was significantly affected by oscillatory motion near

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the critical stall angle. Calculated hysteresis in turbulent separation was found to follow previously described dynamic stall delay and reattachment delay character but the authors' numerical work was otherwise unrewarding.

The authors have relied upon the analytic examination of oscillating airfoils containing the qualitative features of Ham's vortex shedding model but modified by Philippe and Sagner with potential vortices. Empirical data were used to determine the angle of attack at which the theoretical vortex shedding could be assumed to begin. The model does not include dynamic stall which is recognized to be a strong viscous-inviscid interaction phenomenon. The authors note the contributions of Crimi and Reeves (1972) but conclude that the phenomenon cannot be included in the present state of the art in fluid machinery analysis. The final analytic model is summarized in a previous paper by McCroskey (1973). The analysis is valid for laminar flow, and the authors have modeled turbulent boundary-layer flow by incorporating the eddy viscosity formulation of Cebeci et al. (1970) into the time-dependent mean momentum equation as derived by Shamroth and McDonald (1971). The final result is only quasi-steady since empirical data required in the formulation were not modified for unsteady effects.

The authors repeat previous warnings that separation cannot always be interpreted as the point of vanishing wall shear, or flow reversal in unsteady flows; however, they offer no new definition. The authors have found major differences between turbulent and laminar flat

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plate flows. Unsteady effects are manifested in velocity profiles and displacement thickness more than in wall shear for the turbulent case. The kinetic energy models for both eddy viscosity and turbulence appear to break down at high dimensionless frequencies, but both are apparently valid for the range of frequencies considered by the authors. The range of validity seems to be larger for the turbulent kinetic energy model than for the eddy viscosity model.

The authors conclude that the main shortcoming of their thin boundary-layer method is its failure to provide reliable clues about the formation and shedding on an oscillating airfoil of the vortex-like disturbance that distinguishes dynamic stall from static stall. They suggest four possible causes of this analytical deficiency: (1) inadequate treatment of the bubble-bursting phenomenon near the leading edge, (2) incorrect treatment of the turbulent boundary layer just downstream of the bubble, (3) inadequate definition of turbulent separation, and (4) failure to account for interaction between the boundary layer and the unsteady potential flow around the airfoil.

Schneider and Kaplan (1975) have approached the problem of modelling fan dynamic response from an interesting point of view. They have not examined the dynamic physical response of the fan but have accounted for the inertial response excited by varying fan torque. In their computer simulation they rely on the static pressure discharge curve, recomputing the slope as a linear function of shaft speed. The shaft speed is computed by analyzing the dynamic response of a rotating inertial system subject to varying torque loading.

SUMMARY

The review of past research efforts on any topic is a continuing process which has no end. An attempt has been made in this review to provide a survey of literature currently available on the subject of ACV lift system and fan dynamics.

The complete and exact three-dimensional unsteady nonhomogeneous rotational viscous-flow model for an ACV lift fan or compressor does not presently exist. Such a model, when developed, will however certainly contain as contributing features some or all of the following elements:

1. Surge

2. Resonance

3. Limit cycle stall

4. Root-to-tip stall

5. Propagating stall

6. Upstream and downstream stator dynamic interactions

7. Foil laminar and turbulent boundary layers

8. Boundary layer transition

9. Separation

10. Reattachment

11. Laminar bubble formation and bursting

12. Vortex shedding from leading and trailing edges
    of rotating foils

13. Vortex street formation

14. Vortex formation by stators

15.   Nonuniform flow fields including volute boundary layers

16.   Separation delay on moving (pitching and/or heaving) air foils

17.   Fan rotor inertial effects and variable rotational rate

18.   Acoustic and high frequency excitation

19.   Compressibility of working fluid manifested as stiffness and damping

20.   Coanda effects on all elements.

These elements have been touched upon in the reviewed literature, but continued research on all phenomena and especially on the interactions among them must continue prior to the development of a complete fan dynamics model.

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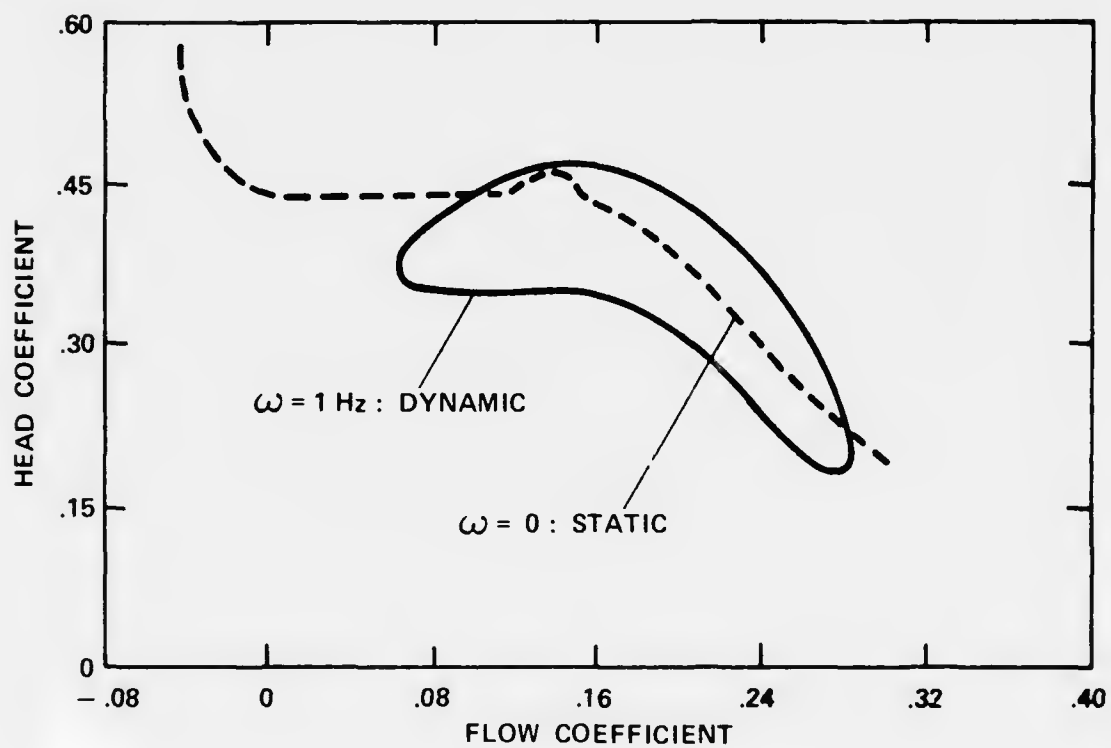
Figure 1 - Dynamic Characteristic of an SES Lift Fan

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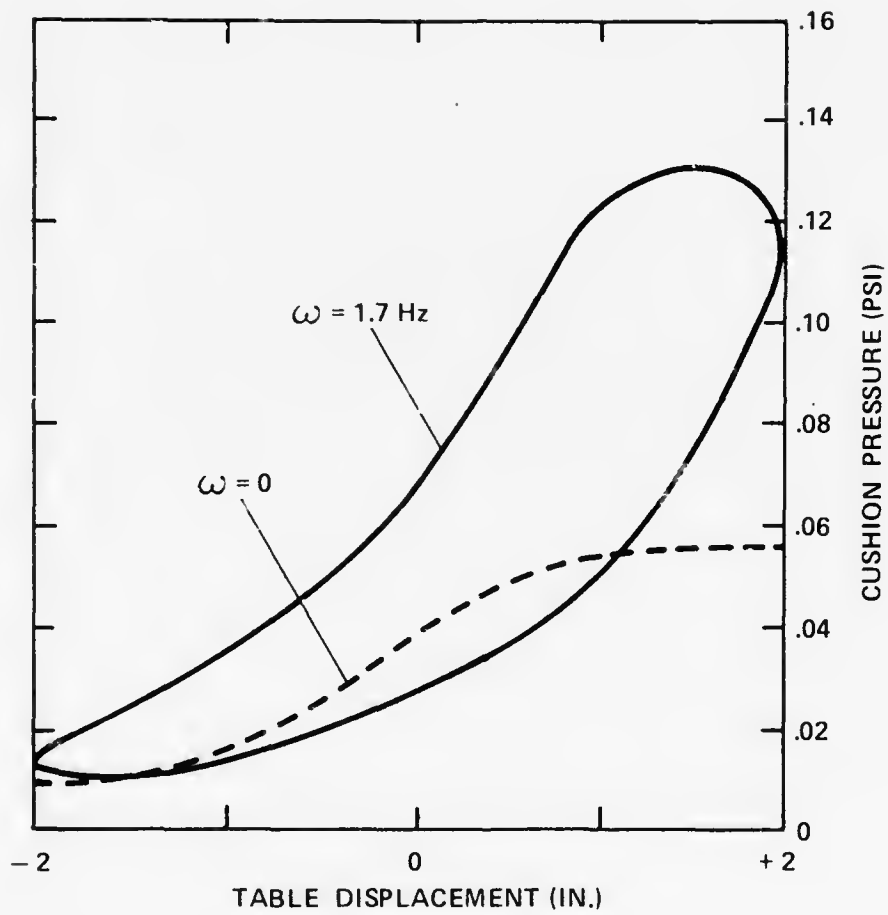
Figure 2 - Dynamic Characteristic of an ACV Lift Fan

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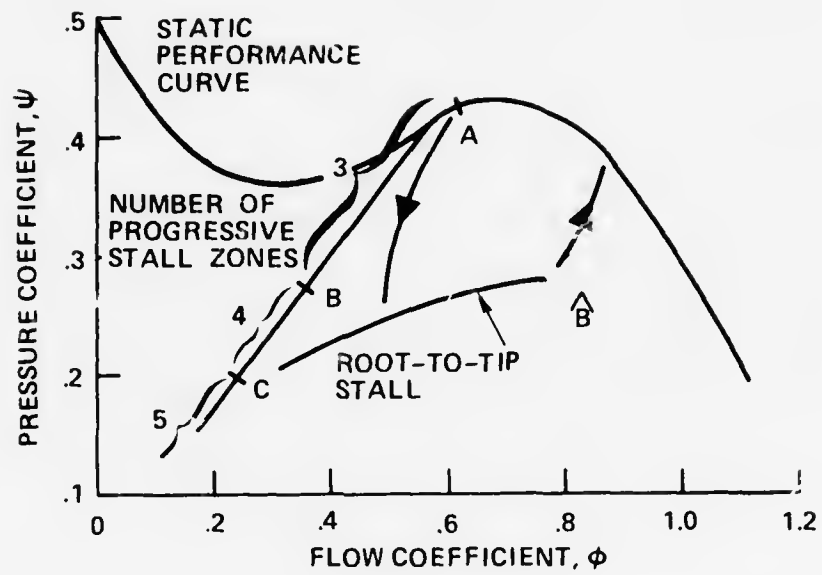
Figure 3 - Pressure Coefficient as a Function of Flow
Coefficient for Progressive Stall and Root-
to-Tip Stall
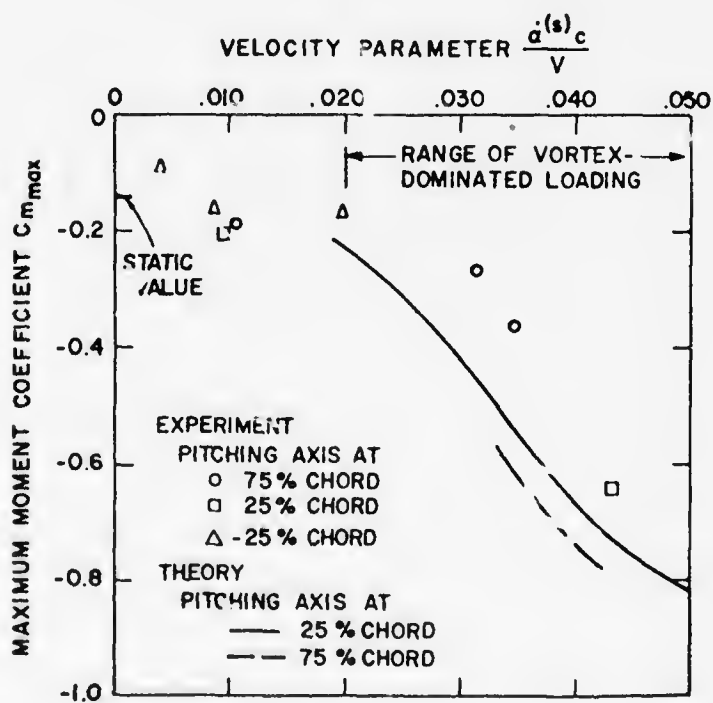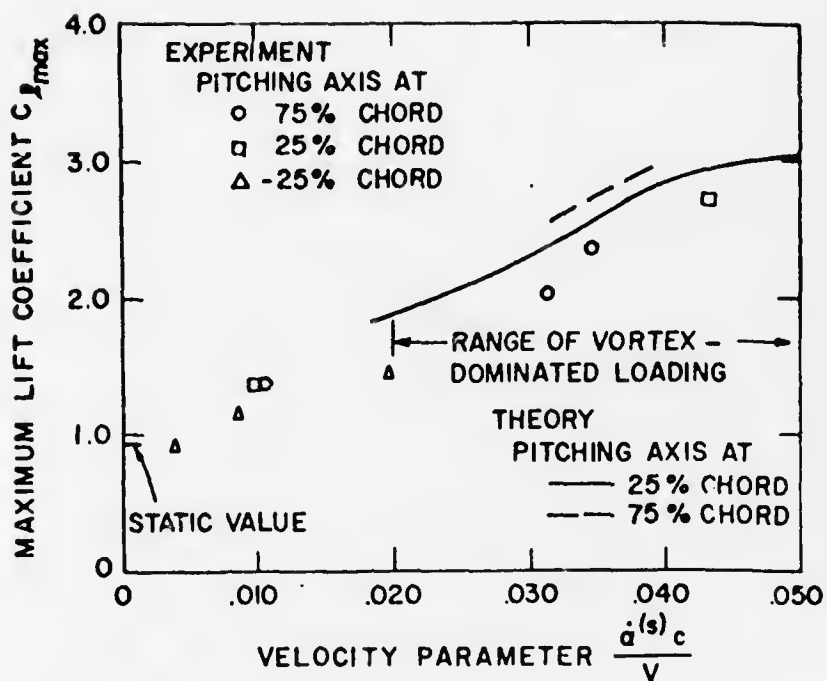
Figure 4 - Maximum Lift and Moment Coefficients as a
Function of Velocity Parameter
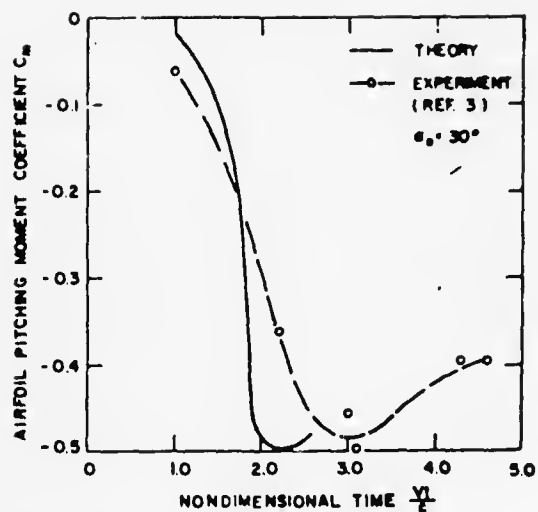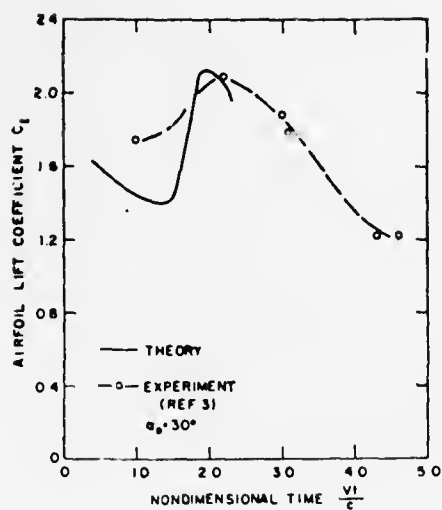
Figure 5 - Airfoil Lift Coefficient and Pitch Moment
Variations Following a Sudden Onset of Flow
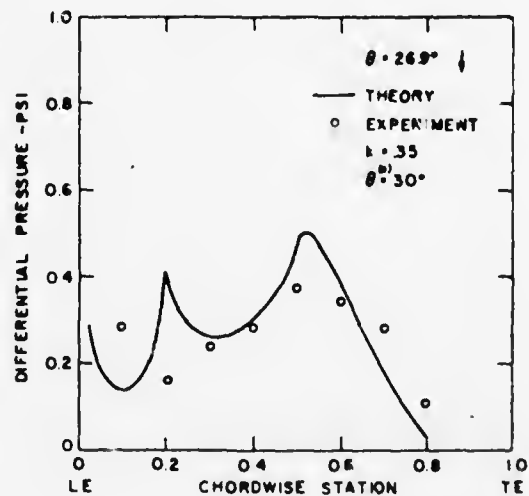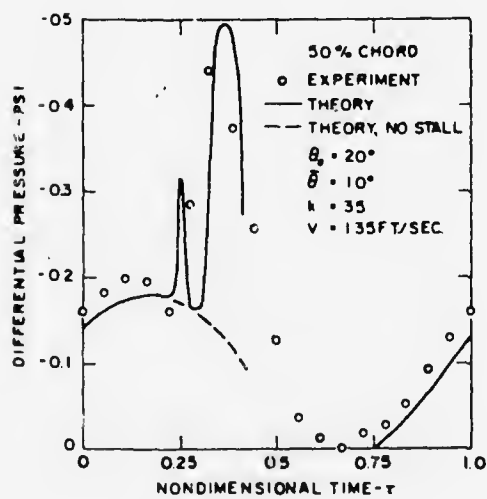
Figure 6 - Temporal and Spatial Unsteady Pressure Distributions

APPENDIX

BIDARD-JENNY ANALYSIS

This analysis is based on that developed by Bidard (1946) and Jenny (]950). The basic analysis was carried out by Bidard in order to study the surge condition. The analysis was then extended by Jenny to allow for a rotary exhaust valve.

As a first approximation the compressor installation is replaced by a model as illustrated in Figure Ala. The point α on the model corresponds to point α on the compressor operating characteristic. Similarly point γ on the model exhaust corresponds to point γ on the exhaust valve characteristic. Under non-steady flow conditions these points α and γ do not usually coincide. At point β on the model the pressure is p' whilst the mass flow is approximately $\dot{m}$. The point β is then located on the pressure-mass flow diagram. In order to study the non-steady flow as a function of time it is necessary to obtain the movement of point β. If this point tends towards the intersection of the characteristic, N, the system is stable, whereas if the point moves away from N the system is unstable. The compressor and exhaust valve characteristics are plotted as a ratio of the operating conditions at the peak of the compressor characteristic, Figure Ala.

As a first approximation Bidard assumed that the mass M is concentrated in one area, and the capacity Q in another, Figure Ala. The air column LA is considered to flow with the velocity c, and to be accelerated or decelerated as a whole by the difference in pressure

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p-p'. The pressure p is taken from the steady flow compressor characteristics and has been made non-dimensional by using the reference point 0. The pressure in the receiver is p', and the mass flow is obtained from the compressor characteristics.

The equation of motion for the gas column is

$$(p-p')A = \rho_m LA\frac{dc}{dt}$$

By substituting
$$\dot{m} = c\rho_m A = c_o \rho_o A$$

$$d\dot{m} \simeq A\rho_m dc$$

and from the gas laws

$$\frac{kp}{a_o^2 \rho_o} = 1$$

then
$$\frac{p}{p_o} - \frac{p'}{p_o} = \left[\frac{d}{dt}\left(\frac{\dot{m}}{\dot{m}_o}\right)\right] T_c k M_o$$

where
$$T_c = \frac{L}{a_o}, \quad M_o = \frac{c_o}{a_o}, \quad B_1 = \frac{1}{kT_c M_o}$$

and

$$\frac{d}{dt}\left(\frac{\dot{m}}{\dot{m}_o}\right) = \frac{1}{kT_c M_o}\left(\frac{p}{p_o} - \frac{p'}{p_o}\right) = B_1\left(\frac{p}{p_o} - \frac{p'}{p_o}\right) \qquad (A1)$$

By considering continuity for the receiver, in which the change is taken isentropically in accordance with Jenny, but at variance with Bidard, the following equation is obtained

$$\frac{d}{dt}\left(\frac{p'}{p_o}\right) = \frac{k\left(\frac{a'}{a_o}\right)^2}{T_Q}\left(\frac{\dot{m}}{\dot{m}_o} - \frac{\dot{m}'}{\dot{m}_o}\right) = \left[\frac{\dot{m}}{\dot{m}_o} - \frac{\dot{m}'}{\dot{m}_o}\right]B_2 \quad (A2)$$

where
$$T_Q = \frac{Q}{Ac_o} \quad \text{and} \quad B_2 = \frac{k\left(\frac{a'}{a_o}\right)^2}{T_Q}$$

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In order to obtain the stability conditions for small disturbances Bidard assumed that the operating point N became the reference point and then replaced the characteristics by the tangents:

$$\frac{p}{p_o} = n\frac{\dot{m}}{\dot{m}_o}, \quad \frac{p'}{p_o} = n'\frac{\dot{m}'}{\dot{m}_o}$$

From equations (1) and (2)

$$\frac{d^2}{dt^2}\left(\frac{p'}{p_o}\right) = B_2\frac{d}{dt}\left(\frac{\dot{m}}{\dot{m}_o}\right) - B_2\frac{d}{dt}\left(\frac{\dot{m}'}{\dot{m}_o}\right)$$

$$= B_2 B_1\left(\frac{p}{p_o} - \frac{p'}{p_o}\right) - B_2\frac{d}{dt}\left(\frac{1}{n'}\frac{p'}{p_o}\right)$$

Therefore

$$\frac{d^2}{dt^2}\left(\frac{p'}{p_o}\right) + \frac{B_2}{n'}\frac{d}{dt}\left(\frac{p'}{p_o}\right) - B_2 B_1\left(n\frac{\dot{m}}{\dot{m}_o} - \frac{p'}{p_o}\right) = 0 \qquad (A3)$$

Now from equation (2)

$$B_2\frac{\dot{m}}{\dot{m}_o} = \frac{d\left(\frac{p'}{p_o}\right)}{dt} + B_2\frac{\dot{m}'}{\dot{m}_o}$$

Therefore, substituting into equation (A3) gives,

$$\frac{d^2}{dt^2}\left(\frac{p'}{p_o}\right) - \left(nB_1 - \frac{B_2}{n'}\right)\frac{d}{dt}\left(\frac{p'}{p_o}\right) + B_2 B_1\frac{p'}{p_o}\left(1 - \frac{n}{n'}\right) = 0 \qquad (A4)$$

Similarly, from equation (A1)

$$\frac{d^2}{dt^2}\left(\frac{\dot{m}}{\dot{m}_o}\right) = B_1\frac{d}{dt}\left(\frac{p}{p_o}\right) - B_1\frac{d}{dt}\left(\frac{p'}{p_o}\right)$$

$$= B_1\frac{d}{dt}\left(n\frac{\dot{m}}{\dot{m}_o}\right) - B_1 B_2\left(\frac{\dot{m}}{\dot{m}_o} - \frac{1}{n'}\frac{p'}{p_o}\right)$$

$$= B_1\frac{d}{dt}\left(n\frac{\dot{m}}{\dot{m}_o}\right) - B_1 B_2\frac{\dot{m}}{\dot{m}_o} + \frac{B_2}{n'}\left(B_1\frac{p}{p_o} - \frac{d\frac{\dot{m}}{\dot{m}_o}}{dt}\right)$$

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therefore

$$\frac{d^2}{dt^2}\left(\frac{\dot{m}}{\dot{m}_o}\right) - \left(nB_1 - \frac{B_2}{n'}\right)\frac{d}{dt}\left(\frac{\dot{m}}{\dot{m}_o}\right) + B_2 B_1 \frac{\dot{m}}{\dot{m}_o}\left(1 - \frac{n}{n'}\right) = 0 \qquad (A5)$$

$\frac{p'}{p_o}$ and $\frac{\dot{m}}{\dot{m}_o}$ define the point β on Figure A1a, and the identical differential equations (A4) and (A5) govern the variation of $\frac{p'}{p_o}$ and $\frac{\dot{m}}{\dot{m}_o}$ with

time. For stability the coefficients of equations (A4) and (A5) must be positive.

Then $\qquad\qquad n' > n, \quad n' < \frac{C}{n}$

where $\qquad\qquad C = \frac{B_2}{B_1} = \frac{k^2 LAM_o^2}{Q}$

This condition for stability is illustrated by Figure A1b. For very large receivers $C \longrightarrow 0$ and stability is only possible for $n \lesssim 0$. The limit is at the peak of the compressor characteristic and the receiver characteristic can have any slope n' in the shaded area. Hence if the compressor operates into a large receiver the surge point will be at the peak of the pressure-mass flow curve; this is generally in agreement with experimental results.

If it is assumed that the receiver characteristic is always positive it is possible to satisfy the stability condition for a positive value of n. At point D, Figure A1b, $n = n' = \sqrt{C}$ and the stability conditions are fulfilled. If n' were greater than $\sqrt{C}$ the stability condition $n' < \frac{C}{n}$ would not be satisfied, and if n' were less than $\sqrt{C}$ the stability condition $n' > n$ would not be satisfied. Hence

point D is the limit of stable operation. At point E the slope n is
less than $\sqrt{C}$ and stable operation is possible provided n' lies in the
shaded area.

Bidard used his basic equations, (A1) and (A2), to calculate
stage by stage the complete surging cycle. Figure A1c is a diagram
taken from Bidard's work showing the cycle of surging obtained by his
graphical solution. If the starting point lies within the stable
region the construction converges on this point showing that all small
disturbances are restored to the equilibrium position. If the starting
point lies in the unstable region the construction builds up into the
surging cycle, the direction of flow is reversed and the receiver is
emptied along the negative branch, the flow is then accelerated and
the receiver filled approximately along the pressure-mass flow curve.

JENNY'S EXTENSION OF BIDARD'S ANALYSIS

Jenny (1950) extended Bidard's work to allow for a rotary
exhaust valve, and to calculate, stage by stage, the non-steady flow
phenomenon. Equations (A1) and (A2) can be written in the form

$$\Delta\left(\frac{\dot{m}}{\dot{m}_o}\right) = B_1 \left(\frac{p}{p_o} - \frac{p'}{p_o}\right) \Delta t \tag{A6}$$

$$\Delta\left(\frac{p'}{p_o}\right) = B_2 \left(\frac{\dot{m}}{\dot{m}_o} - \frac{\dot{m}'}{\dot{m}_o}\right) \Delta t \tag{A7}$$

$\frac{\dot{m}'}{\dot{m}_o}$ is obtained from the steady flow valve characteristics for the

pressure $\frac{p'}{p_o}$, and $\frac{p}{p_o}$ is taken from the compressor characteristic for

the mass flow $\frac{\dot{m}}{\dot{m}_o}$.

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The process of this stage by stage calculation is illustrated in Figure A1d. The rotary valve characteristics are shown so that in each time interval, Δt, the characteristic changes from t_1 to t_2 to t_3, etc. The calculation is commenced at some point 2; the location of this point is either guessed or estimated from experience. In order to locate point 3 the values of $\left[\dfrac{p}{p_o} - \dfrac{p'}{p_o} \right]$ and $\left[\dfrac{\dot{m}}{\dot{m}_o} - \dfrac{\dot{m}'}{\dot{m}_o} \right]$ are obtained from the graph. $\left[\dfrac{p}{p_o} - \dfrac{p'}{p_o} \right]$ is the difference in pressure between the steady flow compressor characteristics and the momentary operating point 2 for the mass flow at 2, and $\left(\dfrac{\dot{m}}{\dot{m}_o} - \dfrac{\dot{m}'}{\dot{m}_o} \right)$ is the difference in mass flow between that at 2 and the mass flow for the valve characteristic t_3 for the pressure at 2. The values of $\Delta\left(\dfrac{\dot{m}}{\dot{m}_o} \right)$ and $\Delta\left(\dfrac{p'}{p_o} \right)$ are then calculated using equations (A1) and (A2). The horizontal distance from 2 to 3 is then equal to $\Delta\left(\dfrac{\dot{m}}{\dot{m}_o} \right)$ and the vertical distance is equal to $\Delta\left(\dfrac{p'}{p_o} \right)$. Thus 3 is located, and similarly round the cycle back to 2. The process is repeated until the cycle is closed. This condition is attained when a calculated point coincides with that obtained during the previous cycle.

The speed at which a calculation settles into a closed cycle depends upon the accuracy of the initial guess, and upon the magnitude of Δt. If the initial guess is perfectly correct only one cycle is necessary. A very poor guess at the starting point can make the calculation extremely long by causing the loop to enter the surge cycle.

In order to avoid this type of difficulty the starting point is usually selected so that the initial loop does not enter the surge cycle. If the calculation does not predict surge it quickly settles into a closed cycle, whereas if surge is predicted the calculation enters the surge cycle after the first loop and then quickly settles into a closed cycle.

Difficulty is also encountered on occasions when the value of Δt is not sufficiently small. When this occurs it is not found possible to obtain a closed cycle. In these cases, in order to perform a satisfactory calculation, it is necessary to reduce Δt to a value less than $\frac{1}{300}$ s .

In order to determine the constants B_1 and B_2 in equations (A6) and (A7), the compressor is replaced by an equivalent pipe using the method suggested by Jenny. The resulting equivalent system contained the same amount of kinetic energy as the compressor. The compressor is 'unwound' and the spiral replaced by a mean length. By integrating the equation of motion through the compressor the length of the equivalent pipe is found, details are given below. For the compressor used in this work the equivalent pipe was found to be 17 in long which, when added to the delivery pipe, gave a total length of 71 in. The constants B_2 and B_1 were defined as:

$$B_2 = \frac{k\left(\frac{a'}{a_o}\right)^2 Ac_o}{Q}, \quad B_1 = \frac{a_o^2}{kLc_o}$$

where A = Cross-sectional area of equivalent pipe.

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L = The total length of the delivery pipe; the equivalent

     pipe plus the length of the delivery pipe.

c_o = Air velocity at the reference condition.

a_o = Reference speed of sound.

a' = Speed of sound at point under consideration.

Q = Total receiver volume.

k = Ratio of specific heats.

By making $\frac{a'}{a_o} = 1$ and substituting the other required values,

B_2 and B_1 were found to be 10·15 and 3470 respectively. The time
interval Δt could be chosen at random, but for ease of calculation
Δt was selected so that each time interval occurred on a known rotary
valve characteristic. For the calculations at 50 c/s eight intervals
per cycle were suitable, giving $\Delta t = \frac{1}{400}$ s. When the frequency was
reduced to 25 c/s it was found that eight intervals per cycle were no
longer suitable as the calculation would not settle into a closed
cycle. In order to overcome this difficulty it was necessary to reduce
Δt by taking 12 intervals per cycle, hence $\Delta t = \frac{1}{300}$ s. For a frequency
of 10 c/s it was found to be difficult to obtain a stable cycle despite
the use of 24 intervals per cycle. It was necessary, therefore, to use
48 intervals per cycle; the intermediate rotary valve characteristics
being obtained by extrapolation from the experimentally known 24 charac-
teristics. This, of course, made the calculations extremely tedious,
and those attempted were only continued until it was known whether the
calculation would or would not predict surge. The results of two

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calculations are given in Figures A2a and A2b.

JENNY'S EQUIVALENT PIPE

Jenny defined an equivalent pipe such that the dynamic be-
haviour of the compressor was similar to that of a column of gas of
length l_{eq}. For this purpose the compressor was 'unwound' and a mean
length replaced the spiral, Figure A3. The equation of motion for
the fluid particle is

$$F - \frac{dp}{ds} A \, \delta s = \delta s \, A \, \left(\frac{\partial c}{\partial t} + c \, \frac{\partial c}{\partial s} \right)$$

where F contains the centrifugal and frictional forces.

$$c = c(s,t) = \text{velocity}$$

$$p = p(s) = \text{density}$$

Then

$$\frac{Z}{\rho} - \frac{1}{\rho} \frac{dp}{ds} = \frac{\partial c}{\partial t} + c \, \frac{\partial c}{\partial s} \tag{A8}$$

where Z is a volume force equal to $F/A \, \delta s$.

For continuity $\quad c \rho A = c_1 \rho_1 A_1$

where suffix 1 refers to the compressor delivery.

Then
$$c = \frac{c_1 \rho_1 A_1}{\rho A}$$

Substituting into equation (A8) and integrating from 0 to 1 gives

$$\int_0^1 \frac{\partial c_1}{\partial t} \frac{\rho_1 A_1}{\rho A} \, ds + \int_0^1 c \, dc = \int_0^1 \frac{Z}{\rho} \, ds - \int_0^1 \frac{1}{\rho} \, dp$$

then
$$l_{eq} = \int_0^1 \frac{\rho_1 A_1}{A \rho} \, ds$$

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For steady flow
$$\frac{\partial c_1}{\partial t} = 0$$

and

$$\underbrace{\frac{c_1^2 - c_0^2}{2}}_{\delta K_{steady}} = \int_0^1 \frac{Z}{\rho}\, ds - \left(\int_0^1 \frac{1}{\rho}\, dp\right)_{steady} \tag{A9}$$

For unsteady flow

$$\frac{\partial c_1}{\partial t}\, l_{eq} = -\delta K + \int_0^1 \frac{Z}{\rho}\, ds - \int_0^1 \frac{1}{\rho}\, dp$$

Assuming that δK is small, $\delta K \simeq \delta K_{steady}$; then substituting for δK from equation (A9) gives

$$\frac{\partial c_1}{\partial t}\, l_{eq} = \left(\int_0^1 \frac{1}{\rho}\, dp\right)_{steady} - \int_0^1 \frac{1}{\rho}\, dp$$

$$\simeq \frac{P_{steady} - P}{\rho_1} \tag{A10}$$

According to equation (A10) the dynamic behaviour of the compressor is similar to that of a column of gas of length l_{eq}. It can easily be shown that the equivalent pipe contains the same kinetic energy as the compressor.

Kinetic energy in the equivalent pipe $= \frac{1}{2} M c_1^2$

therefore   kinetic energy $= \frac{1}{2} A_1 \rho_1 l_{eq} c_1^2$

Now
$$l_{eq} = \int_0^1 \frac{A_1 \rho_1}{A\rho}\, ds$$

therefore,

kinetic energy in the equivalent pipe $= \frac{1}{2} A_1 \rho_1 \int_0^1 \frac{A_1 \rho_1}{A\rho} c_1^2\, ds$

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From continuity
$$c = \frac{c_1 A_1 \rho_1}{A\rho}$$

therefore,

kinetic energy in the equivalent pipe $= \int_0^1 A\rho c^2 \, ds$

which is the kinetic energy in the 'unwound' compressor.

The equivalent length l_{eq} is given by

$$l_{eq} = \int_0^1 \frac{A_1 \rho_1}{A\rho} \, ds = \int_0^1 \frac{c}{c_1} \, ds \quad .$$

This equation was solved by graphically integrating the velocity c throughout the compressor. c is that component of the velocity which is proportional to the quantity of flow.
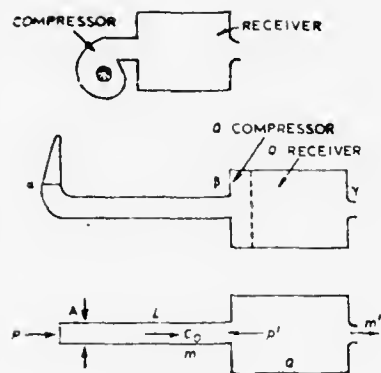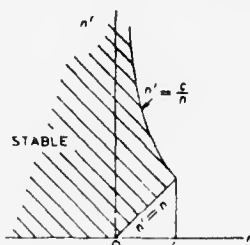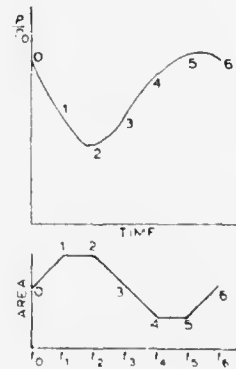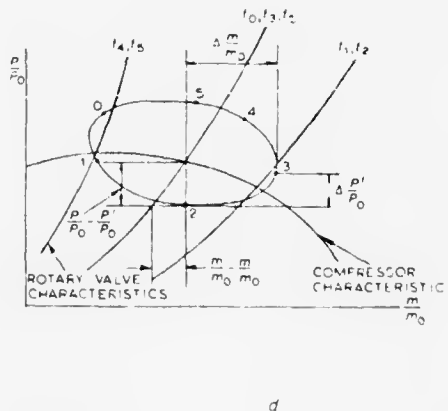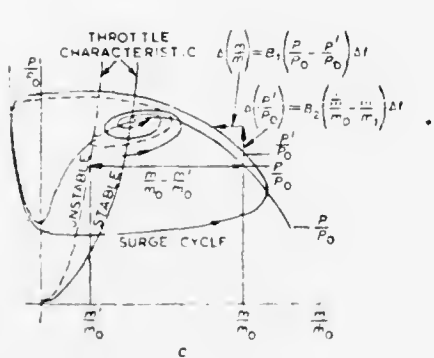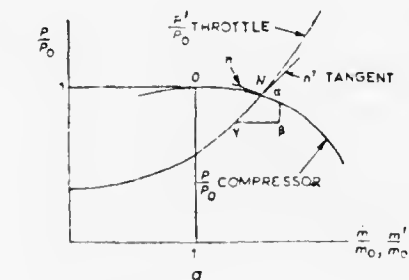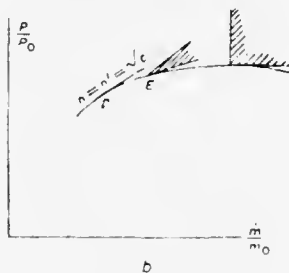
Figure A1 - Bidard-Jenny Calculations
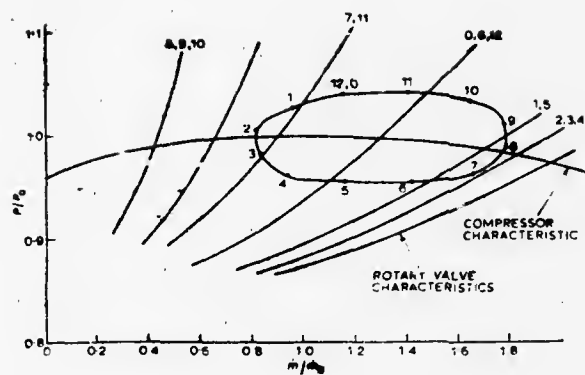
a 50 c/s.
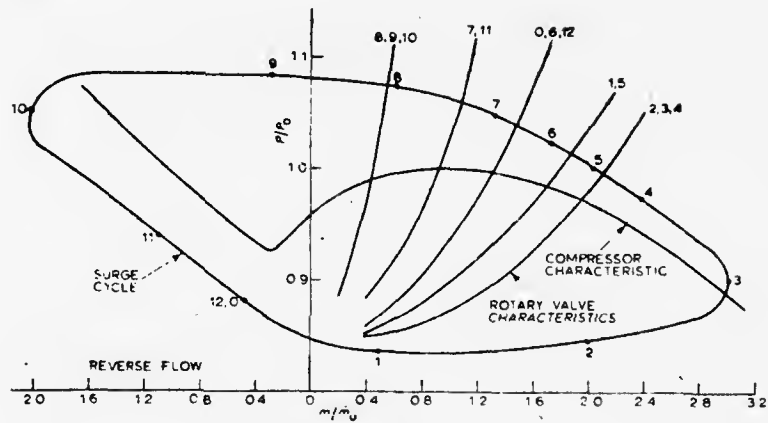
b 25 c/s.

Figure A2 - Unsteady Flow Solution for Exhaust
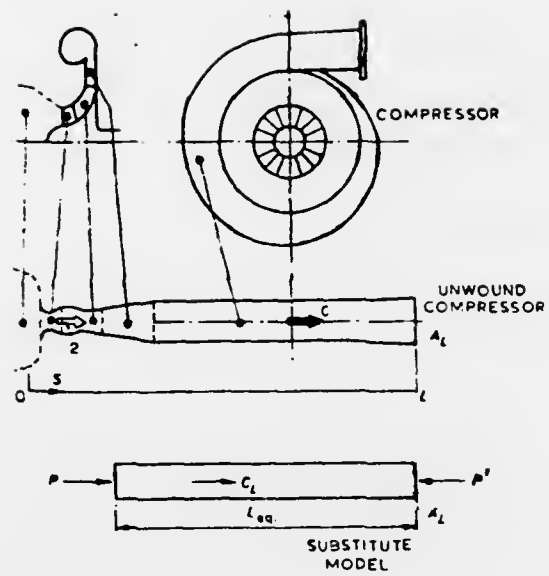Orifice A2 and Nozzle B

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Figure A3 - The Substitute Model

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